DEPARTMENT OF ECONOMICS AND MANAGEMENT

UNIVERSITY OF ECONOMICS IN BRATISLAVA
FACULTY OF ECONOMICS AND FINANCE

THE EFFICIENCY OF LEVERAGED ETF'S FROM INVESTOR'S PERSPECTIVE

Master Thesis in Finance

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Matr. n. 524160

Academic year 2023/2024

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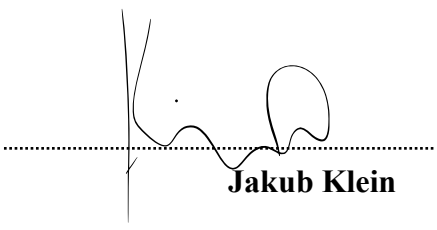
Affidavit

I hereby declare that I am the sole author of this work. No assistance other than that which is permitted has been used. Ideas and quotes taken directly or indirectly from other sources are identified as such.



In Bratislava,

Date: 24.6.2024 **Jakub Klein**

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ABSTRACT

KLEIN, Jakub: *The efficiency of leveraged ETFs from investor's perspective*. [Master Thesis] – University of Economics in Bratislava. Faculty of National Economy; Department of Banking and International Finance. – Thesis Supervisors: doc. Ing. Peter Árendáš PhD., prof. Dennis Marco Montagna – Bratislava: NHF EU, 2024, 88 pages

The diploma thesis is focused on the analysis of the efficiency of Leveraged ETFs from the investor's perspective. The primary goal is to examine the profitability, efficiency and usage of Leveraged ETFs and compare the results with simple ETFs. The work provides explanation and test of created investment strategy. We also examine effective holding periods and behavior of Leveraged ETF in market.

In the theoretical part, the work focuses on the definition of basic terms and principles of Leveraged ETFs and simple ETFs, creation and redemption process and their classification. In addition to benefits and weaknesses, mechanics, and regulation of Leveraged ETFs. In the second part, under research methodology, we describe the procedures that were chosen for data collection and analysis of the topic. The practical part consists of the presentation of the market structure review, examination of pricing, performance, efficiency and holding periods of Leveraged ETFs following by statistical and volatility analysis of ETFs. Subsequently, based on the obtained data and applying the knowledge from theoretical and practical part we propose the trading strategy with usage of Leveraged ETFs.

Through rigorous investigation and observation of the behavior of LETFs, this work aims to contribute to the debate with insights into the profitability, efficiency, and utilization of LETFs. It is hoped that the findings will be helpful for investors in their investment asset selection and investment decisions. The work also contributes to a deeper understanding of the market dynamics of LETFs and ETFs.

Keywords:

ETF, Leveraged ETF, efficiency of Leveraged ETFs, profitability of Leveraged ETFs, effective holding period, investment strategy

ASTRATTO

KLEIN, Jakub: *L'efficienza degli ETF a leva dal punto di vista dell'investitore.* [Tesi di laurea] - Università di Economia di Bratislava. Facoltà di Economia Nazionale; Dipartimento di Banca e Finanza Internazionale. Ing. Peter Árendáš PhD., prof. Dennis Marco Montagna - Bratislava: NHF EU, 2024, 88 p.

Il lavoro di tesi è incentrato sull'analisi dell'efficienza degli ETF a leva nel contesto di portafoglio di un investitore. L'obiettivo principale è esaminare la redditività, l'efficienza e l'utilizzo degli ETF a leva attraverso il confronto con gli ETF a replica passiva. Questa ricerca è divisa in due parte: nella prima si espone un quadro complessivo degli strumenti mentre nella seconda si propone una strategia di investimento. Nell'analisi si da importanza alla disamina su per quanto tempo gli etf rimangono nei portafogli e il comportamento degli ETF a leva sul mercato.

Nella parte teorica, il lavoro si concentra sulla definizione dei termini e dei principi di base degli ETF a leva in confronto a quelli semplici, sul processo di creazione e rimborso e sulla loro classificazione. Inoltre, vengono analizzati i vantaggi e i punti deboli, i meccanismi e la regolamentazione. Nella seconda parte, sotto la voce metodologia di ricerca, vengono descritte le procedure scelte per la raccolta dei dati e l'analisi dell'argomento. La parte pratica/empirica consiste nella presentazione dell'analisi della struttura del mercato, nell'esame dei prezzi, della performance, dell'efficienza e dei periodi di investimento degli ETF a leva e nell'analisi statistica e della volatilità degli ETF. Successivamente, partendo da questi risultati ottenuti e applicando le conoscenze acquisite nella parte teorica e pratica, proponiamo una strategia di trading con l'utilizzo di ETF a leva.

Attraverso un'indagine rigorosa e l'osservazione del comportamento di questi strumenti, il lavoro intende contribuire al dibattito con approfondimenti sulla redditività, l'efficienza e l'utilizzo dei LETF. Lo scopo è che i risultati siano utili agli investitori nella selezione degli asset e nelle decisioni di investimento in quanto questo lavoro contribuisce ad approfondire la comprensione delle dinamiche di questi strumenti.

Parole chiave

ETF, ETF a leva, efficienza degli ETF a leva, redditività degli ETF a leva, period di investimento effettivo, strategia di investimento

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List of Abbreviations

ETFs – Exchange traded funds

LETFs – Leveraged exchange traded funds

ETP – Exchange traded products

ETCs – Exchange traded commodities

ETNs – Exchange traded notes

NAV – Net asset value

DAXX – Deutscher Aktien Index or the GER40

STOXX - Dow Jones Euro STOXX 50

SEC – Securities and Exchange Commission

CFTC –Commodity Futures Trading Commission

ETNs – Exchange traded notes

IPS -Index Participation Shares

SPY - SPDR S&P 500 ETF Trust

SPDR - Standard & Poor’s 500 Depositary Receipt

TIP - Toronto Index Participation

UCITS - Undertakings for collective investments in transferable securities

UYG – ProShares Ultra Financials

FAS - Direxion Daily Financial Bull 3X Shares

SKF – ProSharesUltraShort Financials

FAZ - Direxion Daily Financial Bear 3X Shares

AUM - Assets Under Management

Introduction

In the era of the twenty-first century, investors are still searching for fresh investment opportunities in the financial market. Their primary objective is to increase profit.

Exchange-traded funds (ETFs) are arguably one of the greatest achievements in financial innovation since the development of financial futures. ETFs have gained huge popularity among investors in recent years. Given that they are both collective investment assets, it is evident that they are relatively similar to mutual funds. The primary objective of ETFs is to closely mimic the performance of their benchmark indexes. ETFs and mutual funds differ primarily in the fact, that ETFs are listed on the stock exchange market and can be traded intraday like stocks. ETF pricing can therefore fluctuate during the day. However, mutual funds can only be traded once a day depending on their NAV, which is determined after market is closed. (Deville, 2008)

To maximize the potential of return Leveraged Exchange Traded Funds (LETFs) were created. Leveraged ETFs are relatively new in the world of exchange-traded funds. Leveraged ETFs are tracking the value of an index, group of stocks, or other ETFs. Leveraged ETF is a type of ETF that uses borrowed capital to increase the daily return of an underlying index or benchmark. Leveraged ETFs provide a return that is a multiple of the underlying asset return, with a 2x leveraged ETF aiming to provide twice the daily asset return, while a 3x leveraged ETF aims to triple it. As the popularity of leveraged ETFs is recently rising, studying their efficiency and performance deserves more attention. Despite their potential for higher returns, leveraged ETFs have been the subject of much debate due to their complexity and the risks involved.

This thesis aims to investigate the efficiency of leveraged ETFs, with a focus on the performance, holding periods and potential usage in investment strategies, as well as examination of the advantages and disadvantages of their usage.

The research will add to what we already know about ETFs and shed more light on how investors can use leveraged ETFs to reach their investment goals. The upcoming sections will give an overview of ETFs and leveraged ETFs, look at what previous studies say about how well leveraged ETFs work, and explain the methods used in this research.

1. The current state of the issue at home and abroad

Leveraged ETFs are relatively new type of exchange-traded fund that uses financial derivatives and debt to amplify the returns of an underlying asset. Unlike traditional ETFs that track the securities in their underlying index on a one-to-one basis, leveraged ETFs aim for a leveraged multiplied ratio. Leveraged ETFs are available for most indexes, such as in the US market Nasdaq 100 Index, the Dow Jones Industrial Average Index,S&P 500 Index or in European market DAX Index or STOXX Index.

Leveraged ETFs are often used by traders who want to speculate on an index price or take advantage of the index's short-term momentum. Despite their high-risk, high-cost structure, they are rarely used as long-term investments.

Leveraged ETFs offer indirect access to the financial derivatives, such as options and futures contracts. However, the costs of financial derivates are higher than the costs of traditional ETFs because of the additional expenses associated with derivatives trading.

There are many books and authors writing about the ETFs and about LETFs as well. They are discussing the effectivity, performance, structure, pricing, volatility and advantages or disadvantages. Many authors are trying to invent, explain and apply investment strategies based on various aspects that should provide investors more efficient portfolio diversification.

1.1. Literature review

The fast development of the market and financial instruments as such forced the book authors and researchers to focus even more attention on it. Fevurly (2013) wrote a book that provides a practical guide and comprehensive overview of professionally managed assets, or investments in which all portfolio decisions and rebalancing are delegated to a fund manager or third-party advice service.

The author presented the major categories of professionally managed assets and revealed the best strategies for investing in these instruments. Furthermore, author provided in-depth knowledge and information to investors for selecting the right assets for their portfolios and revealed asset's risks and rewards. Since ETFs had become popular financial security, many people started to examine new types of instruments like LETFs. Guedj et al. (2010) tried to estimate distributions of holding periods for investors in leveraged and inverse ETFs.

The authors estimated the investment shortfall strategy by investing in leveraged and inverse ETFs compared to investing in a simple margin account. Apart from that, the

authors discussed the viability of leveraged and inverse leveraged ETFs and their rebalancing.

Li and Zhao (2014) explained the correlation between trading volume of the LETFs with trading volume of the stocks and the relation between trading of LETFs and price volatility of component stocks. Focusing more on the mathematical analyses, Guo and Leung (2015) wrote a chapter in the book, analyzing the tracking performance of commodity leveraged ETFs and examining associated trading strategies and their performance by back testing with historical price data.

Doubts about the levered ETFs efficiency had made Rompotis (2016) wrote the article which was focusing on the performance and volatility of leveraged ETFs and investing in stock indices from emerging markets. The author considered the sample of various leveraged and inverse levered ETFs that covers country or regional emerging market indices, following by investigation of LETFs short-term and long-term performance, the targeted returns, volatility, persistence in their volatility and spillover effects on returns. Results revealed that, on average, leveraged ETFs can achieve their return targets within a weekly timeframe at most.

Conversely, inverse ETFs tend to reach their return targets within a 2-day period on average. Regarding the risk, the analysis indicates that the volatility of leveraged ETFs closely mirrors that of their targets and remains notably persistent over time.

Providing practices and research with a detailed reference tools for navigation through the market and insight view for making investment decisions, Charupat and Miu (2016), in their article, stated the main concepts and explanations of all important aspects of levered ETFs, focusing on key elements like structure, pricing, performance, regulations, taxation, and trading strategies. Subsequent chapters bridge theoretical concepts with practical applications, exploring mechanics, portfolio rebalancing techniques, and the daily compounding effects that contribute to the attractiveness of investing in these funds.

Even though the world is divided into different market regions Miu et al. (2020) investigated the tracking performance and pricing efficiency of five groups of equity leveraged ETFs traded in Japan by development of framework for determination of theoretical returns, based on the costs of carry of their underlying assets.

Moreover, by using the theoretical framework, authors reconciled the performance behaviors that can be attributed to the heavy reliance on futures contracts. In the study DeVault et al. (2021) explored the expanding role of leveraged ETFs within institutional portfolios. The research delves into the dynamics of this evolving trend over time, focusing

on the increasing significance of LETFs in the overall outlook of institutional investment strategies. Topic of the study focuses on the consideration of diversification strategies and the integration of LETFs into various institutional portfolios.

The authors pointed how LETFs, with their inherent leveraged structures, contribute to diversification efforts and how institutional investors are strategically incorporating them into their portfolios.

The exploration of Leveraged Exchange-Traded Funds has attracted many authors with studies conducted by Trainor Jr., Charupat and Miu (2010). These investigations highlighted on additional aspects of LETFs, ranging from their impact on market volatility to the assessment of pricing efficiency. Focus of the study was to understand the influence of ETFs on the non-fundamental volatility of the securities within their portfolios. This research undertook the task of solving the essence of heightened volatility and inspecting the link between end-of-day price momentum, LETFs, and the consequential impact of rebalancing trades.

Complementing this, another work of Charupat and Miu (2011) explored the area of pricing efficiency associated with LETFs. The authors set out to measure the effectiveness of LETF pricing mechanisms, observing the positive and negative correlations between price deviations of bull and bear funds and the corresponding returns on their underlying indices.

Together, these studies contribute substantially to the continuous debate on LETFs, offering insights into their versatile impact on market dynamics, the difficulties of pricing efficiency, and the interplay between price deviations and underlying index returns. To understand better the issue of understanding implied volatilities from options written on LETFs especially with a focus on the relationships between LETF options with varying leverage ratios.

Leung and Sircar (2014) investigated the empirical data on implied volatility for LETF options based on the S&P500. To enhance the comparison with non-leveraged ETFs, the authors introduced the concept of moneyness scaling and presented a new formula that connects options implied volatilities between leveraged and unleveraged ETFs.

Peterburgsky (2018) presented opinion that significantly contributes to the ongoing discourse on investment strategies involving leveraged and inverse leveraged Exchange-Traded Fund (ETF) pairs. The report suggests that equivalent to a simulation analysis done by Jiang and Peterburgsky (2017), analyzing investment strategies utilizing triple-leveraged and inverse triple-leveraged ETF pairs, simulating daily returns across a span of

48 years. Results indicate that many of these strategies exhibit significant outperformance compared to the S&P 500 when considering risk-adjusted returns. For instance, adopting a strategy of shorting the bear triple-leveraged ETF and the bull triple-leveraged ETF in a 2:1 ratio (while simultaneously holding long positions in Treasuries) resulted in an average annual Sharpe ratio surpassing four times that of the S&P 500.

The major conclusion of the study, consistent with the simulation analysis done earlier, is demonstrating the superior performance of the straightforward portfolios in comparison to the respective underlying index. This outperformance is particularly evident when we evaluate risk-adjusted measures, indicating a potential possibility for investors to achieve higher returns while managing risk.

Over the years various empirical studies have assessed the performance of daily LETFs over extended time periods beyond a single day. The prevailing agreement from these studies suggests that leveraged ETFs effectively mirror the leveraged multiple of their benchmark indexes' returns in the short term. However, over the long term, these ETFs tend to deviate from their expected performance.

Notably, these deviations can be significantly negative, introducing substantial risk for investors holding long positions in leveraged ETFs. In summary, the research by Trainor and Baryla (2008) indicates that a 2x daily leveraged S&P 500 ETF offers a moderate increase in expected return over short time spans (1, 3, 5, and 10 years) but comes with a significant rise in standard deviation.

Wang et al. (2009) suggest that over periods not exceeding one month, 2x and -2x daily leveraged ETFs generally provide returns that are proportional to their leverage ratios concerning the underlying benchmark index. However, deviations occur for time spans longer than one month, attributed in part to the quadratic variation of the benchmark index.

Additionally, Bansal and Marshall (2015) showed that, for investment periods spanning over one year from 1964 to 2013, the average disparity between the return of a leveraged S&P500 ETF and the multiplied product of the underlying benchmark index's return and the leverage amount is consistently greater than zero.

This indicates a clear potential for a daily leveraged S&P 500 ETF to generate a substantial amplification of return over the course of one year. Theoretical analyses often focus on the implications of holding a continuously leveraged ETF in the long term. According to Cheng and Madhavan (2009), when the underlying benchmark index follows a geometric Brownian motion, continuously leveraged ETFs seem to lead to value destruction over an extended period. Jarrow (2010) added that, at a minimum, leveraged ETFs fail to attain their expected leverage multiple in the long-term run. Measuring the

risk associated with leveraged ETFs, Leung and Santoli (2012) provided a measure and described admissible leverage multiples accordingly. This research contributes to the understanding of the challenges and limitations posed by continuously leveraged ETFs in the context of long-term investment strategies.

Contrarily, additional theoretical results delve into long-term positions in an ETF that undergoes discrete leverage adjustments over time. Avellaneda and Zhang (2010) offered an approximation for the long-term return of a daily leveraged ETF, specifically for investment horizons of less than one year. This approximation is grounded in the leverage multiple, as well as the mean and variance of the daily returns of the underlying index. Empirically, the approximation demonstrated high accuracy, particularly for quarterly periods.

During the financial crisis spanning from 2008 to 2009, daily LETFs generally failed to achieve their intended multiple of daily returns, as Shum and Kang (2013) registered. Similar findings were reported by Tang and Xu (2013). These differences can be attributed to factors such as management and trading premiums/discounts, resulting in a reduction in the amplification of daily returns.

For instance, 2x and -2x daily leveraged S&P 500 ETFs behaved more like 1.9x and -1.9x daily leveraged ETFs during the financial crisis. Although the randomness of these errors made them challenging to include into theoretical results. The findings can account for such errors by considering an increased expense ratio. This acknowledgment reflects an attempt to address the impact of management and trading discrepancies on the performance of daily leveraged ETFs during times of financial crisis.

The simulation of 3x and inverse 3x daily S&P 500 LETFs suggested, according to Charupat, Ma and Miu (2022), that the long-term performance of these LETFs is influenced by a combination of volatility and the market condition of the S&P 500 index, whether it is sideways, up-trending, or down-trending.

The theoretical foundation presented in this concept complements and supports the insights derived from the simulation-based findings. This suggests that the performance of LETFs is intricately tied to both market volatility and the directional trends in the underlying index, providing a more comprehensive understanding of the factors influencing their long-term outcomes.

1.2. Definition of Exchange traded funds

On September 26, 2019, the U.S. Securities and Exchange Commission (SEC) granted approval to Rule 6c-11 under the Investment Company Act of 1940, along with the corresponding changes of Form N-1A, referring to exchange-traded funds (ETFs). The ETF Rule, officially known as Rule 6c-11, permits the operation of both index-based and actively managed ETFs, contingent upon adherence to specified conditions outlined in the rule. One notable condition is the requirement for these ETFs, to offer complete transparency of their portfolios.

The ETF Rule allows ETF shares to be traded at secondary market prices, exempting them from the requirement to transact at net asset value (NAV) mandated by Section 22(d) of the 1940 Act. It also provides exemptions for affiliated transactions, permitting individuals with a 5% or greater ownership stake to engage in in-kind purchases and redemptions. Additionally, ETFs are granted a 15-day window, instead of the usual seven days, to fulfil in-kind redemptions involving foreign investments, as per an exemption from Section 22(e) of the 1940 Act.

1.2.1. Classification of ETFs

When categorizing various types of ETFs, the initial approach involves classifying ETFs as a collective entity. Subsequently, the types of ETFs will be enumerated based on their replication strategy and categorized by the underlying asset.

1.2.1.1. ETF as a part of ETP

ETFs are a part of the broader category known as Exchange Traded Products (ETPs) which can be classified into three main groups. The defining features of ETPs are evident in their name, as these are products exclusively traded on exchanges. These financial instruments are designed to track specific underlying assets to varying extents, and in some instances, aim to amplify the returns of the underlying assets. This category includes:

1. Exchange Traded Funds (ETFs)
2. Exchange Traded Commodities (ETCs)
3. Exchange Traded Notes (ETNs).

1.2.1.2. Exchange Traded Commodities

ETCs (exchange-traded commodities) are financial instruments without interest payments that serve as hedging instruments on the financial market.

„ETCs should not be regarded as a very special subcategory of commodity ETFs, i.e. ETFs tracking prices of commodities. Such an approach is incorrect as the features of ETCs differ significantly from commodity ETFs." (Marszk, 2017, p.15)

1.2.1.3. Exchange Traded Notes

Baiden (2011) explains an exchange-traded note (ETN) is a type of senior, unsecured, and unsubordinated debt security issued by an underwriting bank. Like other debt securities, ETNs come with a maturity date and rely solely on the creditworthiness of the issuer for backing.

„ETNs are similar to zero-coupon bonds that are sold in very low denominations with mid to long-term maturities, early redemption clauses, and variable interest rates. Once the notes are issued, they typically trade on major U.S. exchanges." (Diavatopoulos, Felton 2009, p.15)

1.2.2. ETFs vs. Comparable investments

ETFs, ETNs and mutual funds offer investors exposure to the returns of diverse underlying market indexes or strategies.

Nevertheless, some investors might discover that the distinctive features offered by ETFs make them more attractive compared to ETNs or mutual funds.

Table 1: ETFs vs. Comparable investments

	ETFs	ETNs	Mutual Funds
Type of Security	Registered investment company[1]	SEC registered debt security	Registered investment company
Ownership	Equity ownership in underlying assets	Senior, unsecured debt obligations of the issuer	Equity ownership in underlying assets
Primary risk	Market risk	Credit and Market risk	Market risk
Liquidity	Intraday on exchange at market price	Intraday on exchange at market price or daily repurchase by issuer[2]	Daily (close of business) at net asset value
Distributions	Yes	None	Yes
Maturity	Perpetual	Typically, 15 – 30 years	Perpetual
Short sales[3]	Available on both an uptick and downtick, subject to borrowing ability	Available on both an uptick and downtick, subject to borrowing ability	No
Voting rights	Yes	No	Yes

Source: Own elaboration based on SEC.gov, available on:https://www.sec.gov/Archives/edgar/data/19617/000087562608001297/overview.pdf

[1]Some ETFs are also registered as trusts, partnerships or commodity pools and are not registered investment companies.

[2]You may exercise your right to have an issuer repurchase the notes subject to restrictions. Each issuer may require investors to have a minimum number of notes for repurchase and a repurchase fee may apply. Please consult your financial advisor for more information.

[3]With short sales, you risk paying more for the security than you received from its sale.

1.3. Exchange traded funds (ETFs)

„Look up in the sky!...It's a bird!...It's a plane!...No, it's Exchange-Traded Funds! Exchange-traded funds are flying high. Better known by the acronym ETFs, each week new funds are launched on Wall Street exchanges and land in the portfolios of investors across the nation." (Ferri, 2008, p.15)

Exchange traded funds investment asset that can often cause confusion in investors' heads when they are thinking about the functionality. The ordinary person, for instance, usually has knowledge about collective investment funds but, ETFs may represent a new type of investment tools that can be used as an investment.

It cannot be denied that from the phrase Exchange traded funds the term "funds" will likely resonate in our ears as first. Subconsciously, people associate related concepts into familiar wholes and attribute similarity to them. Therefore, we must classify the term ETF and clarify its differences and similarities, between exchange traded note (ETNs) and mutual funds.

Hill at al. (2015, p.2) explain *„Exchange-traded funds provide liquid access to virtually every corner of the financial markets, allowing investors big and small to build institutional-caliber portfolios with management fees significantly lower than those typical of mutual funds. High levels of transparency for both holdings and the investment strategy help investors easily evaluate an ETF's potential returns and risks."*

ETFs are hybrid investment assets that combine elements typical for collective investment funds and characteristics typical for stocks. A feature that is similar to collective investment funds is the purchase of a specific share in an ETF, along with the fact that ETFs are managed by an investment advisor. Another similar characteristic is the regulation by the law.

One notable difference between ETFs and collective investment funds, highlighted by Hill et al., is that ETF shares are continuously traded on the market and exchanges, allowing them to be bought through intermediaries and maintain the liquidity throughout the trading day despite to continuous pricing.

Additionally, ETFs are distinguished by the daily publication of their shares. This feature enables investors to compare the ETF's price with the spot price of the underlying asset or

the value of the share of the underlying asset included in the fund's portfolio. (Hill at el., 2015)

Rejnuš (2014) explains that ETFs are funds composed of assets from multiple asset classes and they are accessible to both institutional and retail investors.
An investor or a security holder has a claim of share of the fund's assets if the event of the fund's liquidation will happen.

Moreover, as Veselá (2019) adds that the price of a particular ETF share is derived from the price of the underlying asset. Therefore, if an ETF replicates the underlying asset, such as stocks, the ETF price does not experience a significant deviation from the underlying asset price (known as tracking error). In the case where an ETF tracks an index with a portfolio containing many underlying assets, including assets with low liquidity from emerging markets, a higher tracking error may occur. Veselá also explains that the liquidity of an ETF is influenced by the liquidity of the underlying asset.

1.3.1. History of Exchange traded funds

ETFs, as we know them today, were introduced in the early 1990s, either in Canada or three years later in the U.S.A.. The concept of trading an entire stock basket in one transaction dates to the late 1970s. In 1989, the American Stock Exchange and the Philadelphia Stock Exchange started trading Index Participation Shares (IPS), but legal issues led to their discontinuation.

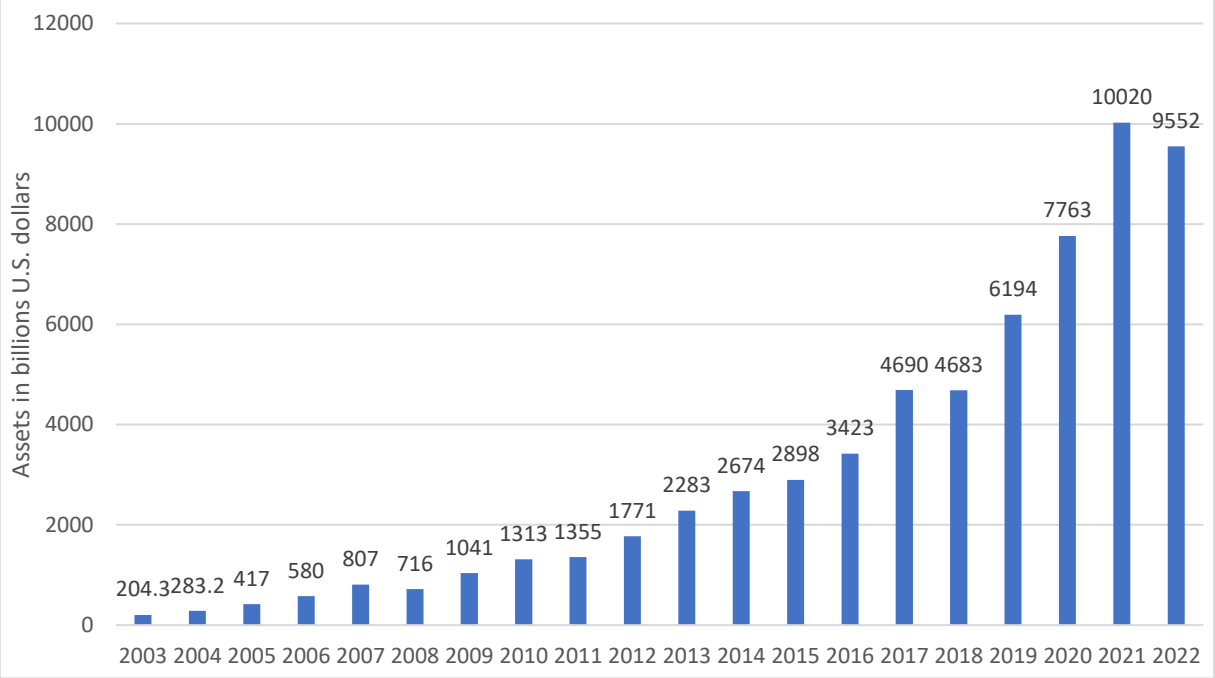
The first equity-like index fund was introduced in 1990, Toronto Index Participation (TIP) that was tracking the Toronto 35. TIP was traded on the stock exchange and featured by exceptionally low management fees, as the fund manager had the authority to lend the stocks held by the fund. TIP was followed by HIP that was based on the wider TSE-100 index, in 1994 and both were terminated in 2000.

According to the Graph 2 we can see total amount of ETFs between years 2003 and 2022. The quantity of ETFs has increased significantly throughout this time, indicating the growing acceptance and appeal of ETFs as investment vehicles.

The increasing range and accessibility of investment alternatives accessible to investors looking to get exposure to different asset classes, industries, and geographical areas is highlighted by the explosion of ETF products. The graph also highlights how the

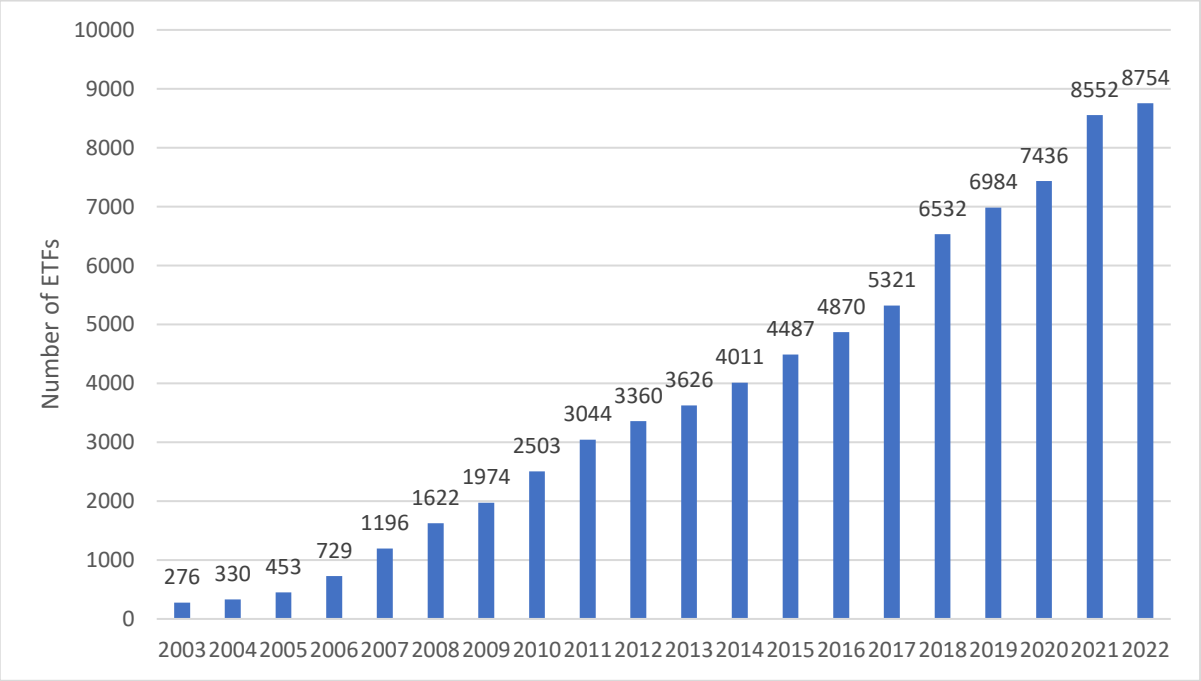
investing environment is changing, with ETFs becoming more and more common in portfolios in both the retail and institutional sectors.

Graph 1: Development of assets of global exchange traded funds (ETFs) from 2003 to 2022 (in billion U.S. dollars)



Source: Own elaboration, based on data form Statista.com (2022)

Graph 2: Number of exchange traded funds (ETFs) worldwide from 2003 to 2022



Source: Own elaboration, based on data form Statista.com (2022)

1.3.2. Benefits of using ETFs as an investment vehicle

An examination of the ETF market should initiate by stating the attributes of ETFs that have led to the notable success of these funds.

1.3.2.1. Lower costs

Most investors favor ETFs primarily for their lower costs. Expense ratios of bond index ETFs averaged 0.12% in 2021, down from 0.26% in 2013. The cost advantage of ETFs stems from their exchange-traded nature, reducing expenses related to recordkeeping and distribution. This contrasts with mutual funds where direct investor-fund company interactions raise overall ownership costs. Overall, ETFs tend to be more cost-effective than traditional mutual funds, making them a preferred choice for many investors.

1.3.2.2. Access

ETFs provide a crucial benefit of wide access, revolutionizing portfolio options by making diverse asset classes like gold, emerging market bonds, and alternative assets accessible to investors. Unlike before, when such investments were awkward and costly for institutional investors, ETFs level the playing field, offering all investors, regardless of size or timeframe, access to a wide range of financial products through their exchange-traded nature.

1.3.2.3. Transparency

Traditional asset management lacks transparency, that could potentially harm investors. Mutual funds disclose portfolios quarterly, with a 60-day lag. Hedge funds and institutional managers' report performance and positions quarterly, leaving investors uninformed between these periods. This lack of transparency can impact investors assets allocation plans. Unlike traditional funds, most ETF providers disclose their portfolios daily, enhancing transparency for investors. Actively managed ETFs, mandated by law, are the most transparent. Overall, ETFs prioritize transparency in portfolio information and naming conventions.

1.3.2.4. Liquidity and price discovery

The fourth significant advantage of ETFs lies in their liquidity. As exchange-traded instruments, ETFs can be traded on secondary markets at different points during the day. They are eligible for margin trading, short selling, options trading, and other activities. Essentially, any action possible with an individual stock can also be executed with an ETF.

1.3.2.5. Tax efficiency and tax fairness

Another significant advantage of ETFs for investors is their tax efficiency, often surpassing that of mutual funds in terms of after-tax returns. This enhanced tax efficiency in ETFs can be attributed to two main factors: lower portfolio turnover and the capability for in-kind redemptions. Index strategies, which form the foundation for many ETFs and some mutual funds, typically involve lower turnover compared to actively managed strategies.

As a result, investors in these index-based ETFs are less exposed to significant capital gains distributions, common occurrence in actively managed mutual funds. Capital gains distributions represent a less-discussed aspect of the mutual fund sector. Annually, numerous mutual funds distribute capital gains to shareholders for various reasons. This may occur when they sell an appreciated stock to generate cash for withdrawal, portfolio rebalancing, or when a held stock is acquired by another firm. Active funds typically distribute these gains to shareholders at the end of the year, requiring investors to pay taxes on the distributed gains. (Hill at el., 2015)

1.3.3. Weaknesses of ETFs

ETF shareholders are facing similar risks to those of other portfolio holders, like mutual funds. Beyond these general risks, there are specific disadvantages and weaknesses that are considered as noteworthy.

1.3.3.1. Fees and bid - ask spread

As indicated by its name, "exchange-traded" implies that the product will exhibit behavior typical of exchanges. Similar to purchasing or selling shares, acquiring or disposing of an ETF security involves a transaction fee.

Ferri (2009) explains that trading ETFs necessitates an intermediary account with a financial institution since ETFs cannot be directly bought. Establishing such an account incurs additional costs for the investor, in addition to the previously mentioned commission fees for individual transactions involving the purchase and sale of ETF securities.

1.3.3.2. Tracking errors

While ETFs are designed to deliver investment outcomes that typically mirror the price and yield performance of their underlying indexes, the trust may encounter challenges in precisely replicating that performance due to trust-related expenses and various factors. This phenomenon is commonly known as "tracking error." Baiden (2011)

Hill et al. (2015) describe tracking error as a metric that indicates the degree of alignment between a portfolio of ETFs. The primary method for identifying this error involves comparing the daily variations between the index and the corresponding tracking fund.

1.3.3.3. Complexity

Ferri (2009) points out the importance of knowledge of the mechanics of ETFs financial investment tool. ETFs combine features from various established investment instruments, and neglecting to understand the principles of flexibility and the functionality of ETFs can lead to misconceptions. This lack of understanding may result in the inappropriate selection of ETFs into a portfolio and failing to deliver the desired returns to investors due to insufficient knowledge.

Ferri (2009) further explains that the lack of information about the ETFs could create confusion between ETFs and other financial instruments and their characteristics. He takes a closer look at the settlement date, where he points out the importance of awareness of the knowledge about the products we trade. The settlement period for ETFs is three days after the trade has taken place, whereas for traditional funds it is usually the next day. This highlights various undesirable situations, that are not in line with investor preferences.

1.3.3.4. Narrow based structures

Approximately more than 90 percent of ETFs are narrow based. Narrow-based ETFs, as per their definition, are linked to specific and limited underlying indices, such as those based on commodities, focused on individual countries, or concentrated on specific sectors. When news of unfavorable market conditions arises, narrow-based ETFs typically experience more pronounced impacts compared to broad-based ETFs. Baiden (2011)

1.3.4. Creation and redemption process of ETFs

ETFs are commonly organized as open-ended companies, allowing flexibility in the number of shares over time. Unlike managed funds, investors must buy or sell ETF shares on a stock exchange rather than directly from the fund. Prior to trading, ETFs go through a creation process in the primary market. They generate large blocks of shares, called "creation units," typically ranging from 25,000 to 200,000 shares.

Mazumder (2014) adds that usually it is a multiple of 50,000 but the amount may sort between 25,000 and 500,000 shares.

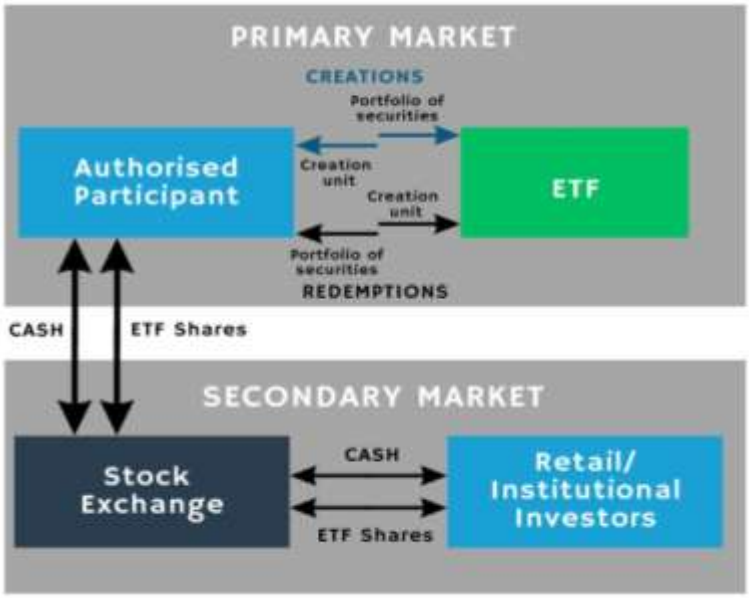
These creation units can only be bought by authorized participants who are often market-makers or registered institutional investors. When acquiring a creation unit for an equity index-tracking ETF, an authorized participant typically exchanges a portfolio of securities with the ETF, rather than using cash. (Kosev and Williams, 2011)

Upon the transfer of the specified creation basket to the ETF, the ETF shares are delivered to the authorized participant. In some cases, the ETF may allow or require the authorized participant to substitute cash for certain assets in the creation basket, especially when acquiring or transferring a specific instrument is challenging or when certain investors cannot hold it. The authorized participant might also be subject to a cash adjustment or transaction fee to offset any incurred transaction expenses. The value of the creation basket, along with any cash adjustment, corresponds to the creation unit's value based on the ETF's NAV at the end of the transaction day. The authorized participant can either retain the ETF shares forming the creation unit or sell them, in whole or in part, to clients or other investors on a stock exchange.

The redemption process is essentially the opposite. A creation unit is redeemed when an authorized participant obtains, through purchases, exchanges, principal transactions, or private transactions, the specified number of shares in the ETF's creation unit and returns

the creation unit to the ETF. In exchange, the authorized participant receives the daily redemption basket, consisting of securities, cash, or other assets. The overall value of the redemption basket aligns with the creation unit's value based on the ETF's NAV at the end of the transaction day. (Antoniewicz and Heinrichs, 2014)

Figure 1: Creation and Redemption process of ETFs



Source: Own elaboration, based on Reserve Bank of Australia (2011), available on https://www.rba.gov.au/publications/bulletin/2011/mar/pdf/bu-0311-8.pdf

1.3.5. ETFs structures

This part gives a quick overview of the main structures used by ETFs and approach the position of leveraged and inverse ETFs in the different divisions.

1.3.5.1. Division by the replication strategy

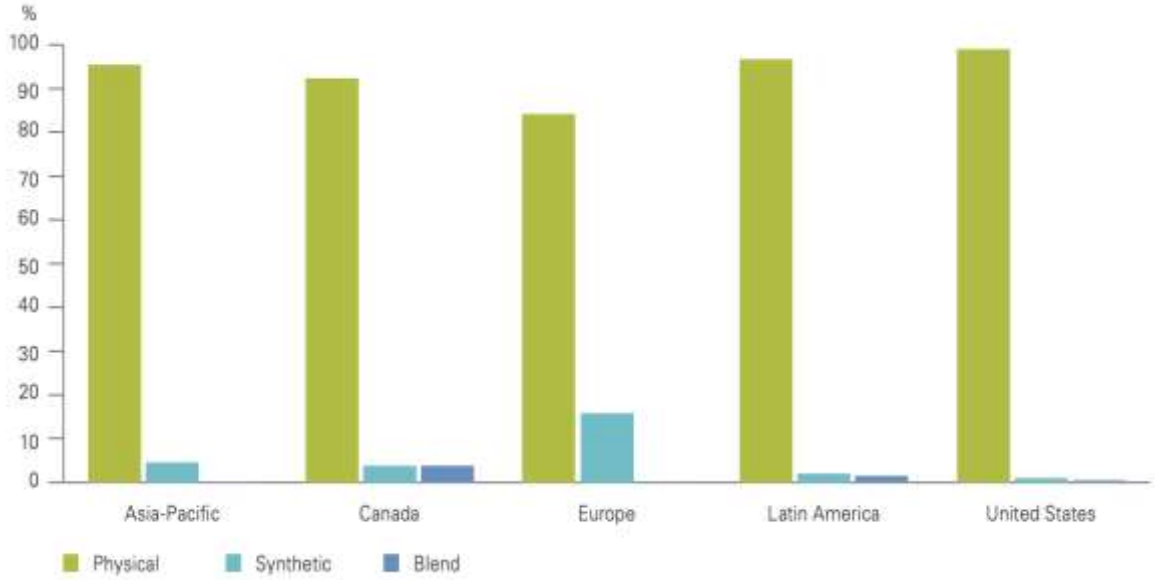
Kosev and Williams (2011) explains that ETFs commonly use two strategies to reach their target returns: physical and synthetic. Physical ETFs own the actual assets that make up a specific benchmark. *„The trading process for ETFs is a distinctive feature that allows them to combine features of traditional open-end funds and those of closed-end funds."* (Pagano et al., 2019, p.10)

For instance, if it's an equity-based ETF, it might hold all or some of the stocks from a benchmark equity index.*„Once the ETF with physical replication is originated, it exchanges "creation units", i.e. a number of ETF shares, with the authorized participants*

(APs) for a basket of underlying securities, the creation basket, or for an equivalent amount of cash that the ETF provider uses to buy the basket." (Pagano et al., 2019, p.11)

The benefits of a physical replication strategy include clear visibility into the ETF's asset holdings and more assurance for investors in case the ETF is closed. In certain regions, especially the United States, regulations limiting the use of derivatives have contributed to the prevalence of physical replication.

Graph 3: Percentage of ETF assets by replication method and region in 2020



Source:Vanguard.com (2020), available on:
https://www.it.vanguard/content/dam/intl/europe/documents/en/physical-and-synthetic-etf-structures-eu-en-pro1.pdf

Corsi et al. (2020) explain that in contrast to physical ETFs, which possess the actual securities of an index, synthetic ETFs gain the returns of an index through swaps.*„Synthetic ETFs replicate the performance of the underlying index with the use of derivatives, which gives rise to counterparty risk."* (Pagano et al., 2019, p.14)

To elaborate, a swap counterparty commits to providing the index's performance for a variable spread, and this spread is paid by the ETF. Synthetic ETFs generally follow two primary approaches: the unfunded model and the fully funded model.

The distinction between the two structures primarily revolves around swap risk and the availability of collateral in the event of a counterparty default. Historically, funded swaps have been perceived as having lower swap risk because they are fully collateralized, and in some cases, over-collateralized. Nevertheless, numerous issuers employing unfunded

models have surpassed the 90% collateral threshold required by UCITS. Unfunded funds also have direct access to the substitute basket, facilitating a prompt liquidation response in the event of a default.*„Derivative-based ETFs comprise leveraged, inverse and other structured ETFs and can be characterized by the intensive use of derivatives as the main financial instruments."* (Pagano et al., 2019, p.15)

In conclusion, it can be summarized that the main factor is the fact whether the ETF's replication strategy consists of replicating the performance of the index or outperforming it.

From this perspective, leveraged ETFs can be categorized as ETFs that seek to achieve multiple returns over and above those of the underlying index or other asset. Of the above divisions, leveraged ETFs are best characterized by the group of derivative-based ETFs.

Assuming that replication of the underlying index (in full, partial and synthetic replication) is a typical feature, leveraged ETFs can be classified as ETFs with special characteristics, whereby special characteristic is meant as generation of multiple of the underlying asset returns.

1.3.5.2. Division of ETFs by asset class

Despite to the aim of the thesis, only a listing of the types of ETFs is provided. The different types of ETFs are not characterized in detail. Directly on the justETF.com website (2023) it is possible to see various groups of ETFs according to several characteristics.

According to the group of underlying assets, bond ETFs, commodity ETFs, equity ETFs, money market ETFs, multi-assets ETFs precious metals ETFs, real estate ETFs and newly created Bitcoin ETFs can be distinguished.

Hill et al. (2015) list the basic types of ETFs as:

- Equity ETFs
- Fixed-Income ETFs
- Commodity ETFs
- Currency ETFs
- Alternative ETFs
- Leveraged and Inverse ETFs

Abner (2016) discusses leveraged and inverse ETFs, highlighting their significant attention in the asset management realm due to the substantial assets they oversee. He

emphasizes their distinct characteristics that contribute to their profitability. It can be inferred from his discussion that Abner does not categorize them as a specific type of ETF with a designated underlying asset, as these funds predominantly derive their value from indices, stocks, or commodities.

It can be observed that Abner (2016) and justETF.com (2023) both mention leveraged and inverse ETFs as ETFs that have exceptional investment style rather than as ETFs set apart within a particular underlying asset, although financial derivatives are playing a big role in their creation, which could confuse not experienced investors.

1.4. Leveraged ETFs

This chapter describes the LETFs. Leveraged Exchange Traded Funds (LETFs) were listed on the market in 2006 by ProShares, although leveraged mutual funds had been already available since 1993. ProShares initially introduced 2x products, and later, in late 2008, Direxion increased the leverage factor with 3x funds.

The key characteristic of LETFs is their aim to provide a multiple of daily returns. Gastineau (2010) suggests that while this instrument can be attractive for investors, it may also lead to disappointment if they are not properly understood.

However, the consistent daily leverage introduces uncertainty in the realized leverage over extended periods. Typically, realized leverage decreases over time due to return volatility. (Avenllaneda and Zhang, 2010)

1.4.1. Importance of Leverage

LETFs, commonly known as leveraged ETFs, represent a distinct category of ETFs designed to generate returns that are more responsive to market fluctuations compared to non-leveraged ETFs.

These funds come in two varieties: bull and bear. A leveraged bull ETF seeks to attain a daily return that is 2 or 3 times the daily return of the underlying index. As an example, we can use ProShares Ultra Financials ETF (UYG) that is offering 2x of the Dow Jones U.S. Financials Index. On the other hand, Direxion Daily Financial Bull 3x Shares (FAS) offers 3x of the same index.

Graph 4: Performance of ProShares UYG and DJI (2015 – 2024)

Source: Tradingview.com (2024)

Graph 5: Performance of Direxion FAS and DJI (2015 – 2024)

Source: Tradingview.com (2024)

Conversely, a leveraged inverse (bear) ETF seeks to attain returns that are -2x or -3x the daily return of the index. This implies that it aims to gain twice or thrice the amount of the market's loss. For example, the ProShares UltraShort Financial ETF (SKF) provides -2x the Dow Jones U.S. Financials Index, while the Direxion Daily Financial Bear 3X Shares (FAZ) tracks -3 times the same index.

Graph 6: Performance of ProShares SKF and DJI (2015 – 2024)

Source: Tradingview.com (2024)

Graph 7: Performance of Direxion FAZ and DJI (2015 – 2024)

Source: Tradingview.com (2024)

For traders operating under Regulation T[4] margin rules, these instruments offer a straightforward means of doubling or tripling their exposure to an index without increasing their capital investment. Additionally, active traders can employ an inverse leveraged ETF as an alternative to short-selling underlying assets, especially when those assets are

[4]Regulation T is a Federal Reserve Board provision that regulates extensions of credit and requires that investors have a minimum initial ownership interest of 50%.

challenging to borrow. As an example, during the latter part of 2008, numerous traders opted for long positions in SKF, a bearish financial fund, as short-selling financial stocks was proving difficult or even impossible. (Avenllaneda and Zhang, 2010)

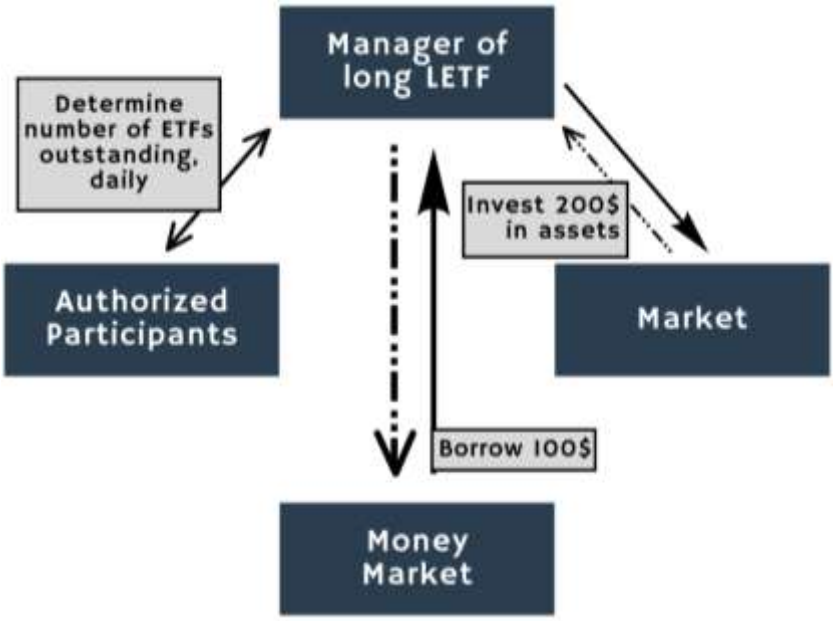
1.5. Mechanics of LETFs

LETFs are funds that can generate multiplied returns (but also loss) than the return of the underlying asset they track. As the name implies, they achieve these multiple returns through leverage. As several authors have stated, the most common LETFs to be encountered are those that have double or triple leverage.

Avellaneda and Zhang (2010) explain that traditional ETFs operate on a one-for-one basis, passively tracking an index or a basket of assets. They are essentially managed without active intervention. In contrast, Leveraged ETFs (LETFs) require active management strategies. This involves activities such as borrowing funds to acquire additional shares for
bullish LETFs or engaging in short selling for bearish LETFs. The position is then rebalanced on a daily basis.

To simplify the hedging process of LETFs, managers often employ daily resetting of total-return swaps with qualified counterparties. Figures 2 and 3 provide graphical representations illustrating the management approach of leveraged ETFs.

Figure 2: "Bullish" Leveraged ETF



Source: Own elaboration, based on Avellaneda and Zhang (2010)

Figure 2 illustrates the management process of a bullish 2x leveraged ETF in a schematic manner. In this scenario, the manager monitors the creation or redemption of new shares. Using this information, the manager makes daily adjustments to the exposure of the ETF to the underlying index by employing leverage.

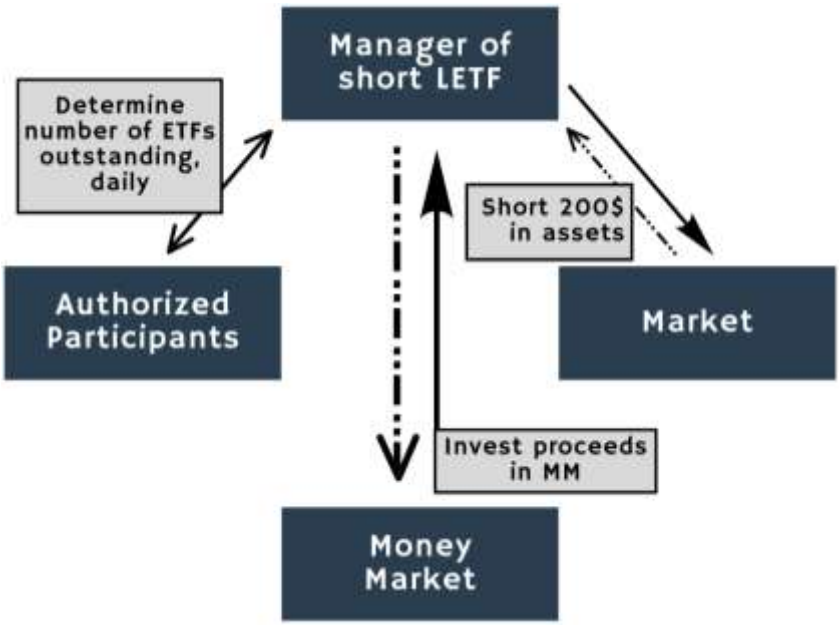
According to Gastineau (2010), Leveraged ETFs utilize financial derivative instruments, primarily swaps and futures contracts, with the primary goal of generating leveraged returns. In the U.S. market, a significant portion, typically 85 to 90%, of LETF assets are invested in shares of major companies by the end of the trading day. These companies are part of the benchmark index tracked by the LETF. The remaining 10 to 15% of assets serve as collateral for futures or swaps, aiming to hedge the leveraged return for the subsequent trading day (Abner, 2016). Figure 3 also illustrates a similar scenario where 85% of the funds are allocated to assets, and 15% are dedicated to derivatives. In this case, the allocation of derivatives involves a split between futures and swaps, with futures providing 25 million USD and 110% exposure, while the remaining 90% exposure is achieved using swaps.

Figure 3: S&P 500 Long Fund

Source: ProShares (2018), available on: https://www.profunds.com/globalassets/proshares/pdfs/strategies/components_of_leveraged_and_inverse_funds.pdf

Several authors, including Abner (2016), extensively delve into discussions about derivative instruments, with a notable focus on futures and swaps. However, more diversity can be observed in the composition of these funds. Charupat and Miu (2011) in their analysis of LETFs in the Canadian market notes the usage of forwards, Baker et al. (2015) take options into consideration. Given the nature and trading characteristics of swaps and forwards, the question of issuer risk becomes apparent. But Hill et al. (2015) explain that ETFs incorporate specific terms and conditions in their contracts, facilitating settlement on the following day. To mitigate issuer risk, these funds implement specialized contractual arrangements. ProShares, Direxion, Powershares, Barclays iPath can be identified as major issuers (Baker et al., 2015).

Figure 4: "Bearish" Leveraged ETF



Source: Own elaboration, based on Avellaneda and Zhang (2010)

Figure 4 illustrates a schematic representation detailing the management approach for a 2x short fund, which is a bearish leveraged ETF. In this specific instance, the manager engages in daily adjustments of the position by implementing double leverage through short selling the underlying asset.

Negative leverage multiples are provided by certain ETFs available in the market. These ETFs are commonly known as short ETFs, bear ETFs, or more specifically, inverse ETFs (IETFs). In the case of IETFs with a leverage of -2x, they aim to deliver a return of 2% in the event of a 1% market decline.

Consequently, these instruments prove advantageous in situations where the market is experiencing a downturn. (Baker et. al., 2015) As Kosev and Williams (2011) stated, Inverse ETFs are also suitable as hedging instruments for existing positions within a portfolio.

Figure 5: S&P 500 Short Fund

Source: ProShares (2018), available on: https://www.profunds.com/globalassets/proshares/pdfs/strategies/components_of_leveraged_and_inverse_funds.pdf

The illustration in figure 5 demonstrates a single approach for achieving a -2x exposure to index. The short fund could hold a substantial portion of its assets in cash. To achieve approximately 30% of inverse index exposure, the fund might utilize some of its cash to initiate short positions on S&P 500 futures contracts.

Additionally, the fund could employ short equity index swap agreements linked to the S&P 500, with a notional value of $170 million. In this arrangement, the fund pays the total return of the index on the $170 million notional value and receives interest payments on the same amount. Both the daily index return (whether positive or negative) and the interest payments would be accounted for on a daily basis.

1.5.1. Volatility and performance of LETFs

According to Sullivan (2009), volatility in the underlying indexes has a significant impact on the returns of leveraged and inverse ETFs. He emphasizes that changes in the pricing of underlying assets have a major effect on the cumulative returns of leveraged ETFs. Furthermore, investors tend to earn less wealth and cumulative return over time when volatility rises. Sullivan adds that as the leverage ratio, or multiplier, rises, the detrimental effect of market volatility on the returns of leveraged ETFs gets worse.

In summary, we can say that while indexes may not always get twice the market return, investors in leveraged exchange-traded funds (ETFs) are exposed to at least twice the risk of the market. Thus, it is crucial for investors to keep a close eye on their leveraged ETF holdings to make sure they are in line with their risk tolerance and investing goals, especially during erratic market conditions.

1.6. Regulations and taxations of LETFs

The regulatory framework governing LETFs is rooted in the Investment Company Act of 1940 and the Securities Act of 1933. LETFs are structured as open-ended investment assets that undergo a registration process with the Securities and Exchange Commission (SEC) under the law, mirroring procedures that are applicable to traditional mutual funds.

Despite many similarities with mutual funds in the form of registration and financial reporting, LETFs have unique characteristics that require an exemption from certain provisions of law before publication.

Notable examples of such relief include permissions for the creation and redemption of "units," allowing authorized participants to trade in predetermined lot sizes, and permitting LETF shares to trade on exchanges at prices different from their net asset values. The SEC, when granting exemptive relief, imposes conditions aimed at enhancing pricing efficiency and safeguarding investor interests. These conditions include regular disclosure of intraday NAVs of LETFs, daily disclosure of closing net asset values, market prices, and premium/discount information.

Regarding the taxation of LETFs, similarly with traditional ETFs the income generated is not taxed at the fund level. However, there are distinctions in tax efficiency due to the way LETFs achieve leveraged returns, primarily through derivatives like swaps rather than actual borrowing and direct investment in underlying indices.

Unlike traditional ETFs, LETFs utilize an “in-cash” redemption process instead of an “in-kind” process because only a portion of their total exposure is in the underlying securities. In this process, authorized participants are redeeming shares and receiving cash equivalent to the redeemed shares.

The in-cash redemption process requires LETFs to generate cash each time there is a net redemption from authorized participants. This can be sourced from their near-cash investments or by selling their holdings of underlying securities. Consequently, LETFs may be involved more in selling activities compared to traditional ETFs, increasing the likelihood of generating capital gains that are subsequently passed on to investors.

Two remarkable facts emerge from this tax structure. First, because LETFs do not have a substantive process, they cannot dispose of securities with a low-price basis, which can result in substantial capital gains when such securities are sold. Second, because many LETFs hold near-cash investments that generate interest income, this income is passed on to investors. (Charupat and Miu, 2016)

2. Aim of the work and Methodology

Leveraged ETFs are one of the most recent innovations in the ETF markets. The aim of this thesis was to investigate the efficiency of leveraged ETFs from investor's perspective. In order of effective examination of the main objective, we settled down several sub-objectives that will allow us to understand this complex topic in detail.

To achieve the main objective, we have set these sub-objectives:

- To compose market overview for better understanding of current trends and future potential of Leveraged ETFs
- To stress out the comparison with classic ETFs within the same market conditions.
- To examine how in leveraged exchange-traded funds work, and how holding period affects the returns and efficiency.
- To explore possibility of usage of leveraged ETFs in the trading strategy and test its efficiency.
- To discuss the advantages and disadvantages of leveraged ETFs and various ways of utilization as investment vehicle.

Our research was primarily focused on investigation of efficiency of LETF within various holding periods and examine usage if chosen LETFs in created trading strategy. Firstly, it was necessary to select specific ETFs and LETFs that follows the same underlying assets. In our work, we chose to focus only on the stock market in the United States. This was due to fact that most of LETFs are issued in Untied States and despite to fact that the US stock market is the largest and most liquid in the world, with the largest share of the ETFs. For our analyses we have chosen ETFs and LETFs shown in Tables 2 and 3.

Table 2: Selected ETFs

Ticker	Name of ETF	Underlying index
SPY	SPDR S&P 500 Trust ETF Trust ETF	S&P 500 Index
DJI	SPDR Dow Jones Industrial Average ETF Trust	Dow Jones Industrial Average
QQQ	Invesco QQQ Trust	NASDAQ 100

Source: Own elaboration

Table 3: Selected LETFs

Ticker	Name of LETF	Underlying index
3USL	S&P 500 3X Daily Leveraged	S&P 500 Index
3USS	S&P 500 3X Daily Short	S&P 500 Index
UDOW	UltraPro Dow30 3x Shares	Dow Jones Industrial Average
SDOW	UltraPro Short Dow30 -3x Shares	Dow Jones Industrial Average
TQQQ	UltraPro QQQ 3x Shares	NASDAQ 100
SQQQ	UltraPro Short QQQ -3x Shares	NASDAQ 100

Source: Own elaboration

The data analyzed in the practical part of the thesis consist of the opening and closing daily trading prices of the assets listed in the Tables above. To obtain good data quality for comparable results we used data all the data from same source Investing.com. For other important data we used available sources such as websites of Statista or BlackRock. The data have been processed and analyzed in the Excel.

The data cover period of the last the last 5 years. Thus, the time interval under study was from 1.3.2019 to 1.3.2024. On this time with obtained data, we have provided the analysis of returns, statistical analysis, computation of Beta coefficients that were calculated to examine the relationship between asset returns, 90 days volatility analysis and performance analysis of the pairs tested by constructed model.

To obtain comparable results, we used regression models and matrix multiplication. Constructed model was based on the arbitrage pair trading and under given trading rules we have investigated and compared its performance of trading different pairs.

Regarding the work with historical data, it should be noted that the calculated return, volatility correlation will be a historical value.

3. Results of the work and discussion

Chapter number 3 focuses on the empirical analyses. In the beginning we have done market overview, that is followed by practical example and analyses of LETFs returns. In the next subchapter we have discussed computation of the impact of rebalancing and compounding on LETFs effective holding period.

After that we have invented an investment strategy and constructed a model where we have used LETFs as an investment asset. We were investigating and analyzing the results we got from the model. Firstly, we have conducted statistical analysis of the S&P 500 (SPY), Dow Jones Industrial Average (DJI), and NASDAQ Composite Index (QQQ) for a given period as well as volatility analysis of the indexes.

Secondly, we have provided statistical analysis, which was followed by correlation analysis and regression model, and in conclusion we analyzed the results we have obtained. With given results we have constructed model focused on the application side of statistical arbitrage method. The model was constructed for underlying indexes, and as well for LETFs, where we were investigating the efficiency and profitability of the created strategy with LETFs in comparison to the same strategy for underlying indexes. At the end of the chapter we have analyzed the results, the causes of the results, the consequences, and possible improvements to the created strategy.

3.1. Market of LETFs

The financial markets have paid close attention to the spread of leveraged exchange-traded funds (ETFs) in recent years. With leveraged exchange-traded funds (ETFs), investors can increase their exposure to a wider range of asset classes, which may result in higher returns but also higher risk.

This analysis critically assesses the state of leveraged exchange-traded funds (ETFs) considering investor behaviour, market dynamics, and regulatory factors.

3.1.1. Market structure

The geographical distribution, asset type and other characteristics of the market structure of leveraged exchange-traded funds (ETFs) in 2023 are noteworthy.

Data from iShares.com[5] covers that, leveraged exchange-traded funds (ETFs) made up roughly 40% of all trading volume in the United States. In the US, 12.7% of equities

[5]iShares.com [electronic source]. online. Available on: https://www.ishares.com/us/insights/global-etf-facts

assets are made up of ETFs; in Europe, 8.5% are, and in Asia-Pacific, 4.4% are. ETFs have a lower market share in fixed income, where they make up 2.6% of assets in the US, 1.8% in Europe, and 0.4% in Asia-Pacific.

In the Table 4 is given the summary of leveraged exchange-traded funds, which also includes average trading volumes, total assets under management (AUM), leverage ratios, and LETF names with their tickers. It displays a selection of 3x leveraged exchange-traded funds covering multiple industries and asset classes, accommodating varying risk tolerances and investment goals.

Table 4: List of 3x Leveraged ETFs (April 2024)

Symbol	ETF Name	Leverage	Total Assets ($MM)	Avg Volume
TQQQ	ProSharesUltraPro QQQ	3x	$22,022.90	72 147 056
SOXL	Direxion Daily Semiconductor Bull 3x Shares	3x	$10,800.80	71 415 032
TMF	Direxion Daily 20+ Year Treasury Bull 3X Shares	3x	$4,632.72	8 587 036
SPYL	Direxion Daily S&P 500 Bull 3X Shares	3x	$4,203.42	7 111 042
FNGU	MicroSectors FANG+™ Index 3X Leveraged ETN	3x	$4,198.49	1 222 206
UPRO	ProSharesUltraPro S&P500	3x	$3,403.32	6 690 583
TECL	Direxion Daily Technology Bull 3X Shares	3x	$3,205.86	2 208 440
NRGU	MicroSectors U.S. Big Oil Index 3X Leveraged ETN	3x	$2,526.64	46 297
FAS	Direxion Daily Financial Bull 3X Shares	3x	$2,370.98	761 919
TNA	Direxion Daily Small Cap Bull 3X Shares	3x	$2,080.41	22 254 346
BULZ	MicroSectorsSolactive FANG & Innovation 3X Leveraged ETN	3x	$1,203.82	183 516
LABU	Direxion Daily S&P Biotech Bull 3x Shares	3x	$1,065.41	3 099 411
YINN	Direxion Daily FTSE China Bull 3X Shares	3x	$1,021.72	7 975 327
UDOW	ProSharesUltraPro Dow30	3x	$644.43	3 162 141
DPST	Direxion Daily Regional Banks Bull 3X Shares	3x	$630.74	1 445 151
URTY	ProSharesUltraPro Russell2000	3x	$523.20	1 482 563
NAIL	Direxion Daily Homebuilders & Supplies Bull 3X Shares	3x	$296.58	219 679
BNKU	MicroSectors U.S. Big Banks Index 3X Leveraged ETNs	3x	$295.90	489 724
GDXU	MicroSectors Gold Miners 3X Leveraged ETN	3x	$273.36	1 265 340
CURE	Direxion Daily Healthcare Bull 3x Shares	3x	$197.73	45 327
WEBL	Direxion Daily Dow Jones Internet Bull 3X Shares	3x	$184.20	791 602
DFEN	Direxion Daily Aerospace & Defense Bull 3X Shares	3x	$157.89	219 586
MIDU	Direxion Daily Mid Cap Bull 3X Shares	3x	$87.21	69 443
EDC	Direxion Daily MSCI Emerging Markets Bull 3x Shares	3x	$82.43	82 503
OILU	MicroSectors Oil & Gas Exp. & Prod. 3x Leveraged ETN	3x	$75.14	137 306
DRN	Direxion Daily Real Estate Bull 3x Shares	3x	$64.21	868 03

HIBL	Direxion Daily S&P 500 High Beta Bull 3X Shares	3x	$61.90	112 54
RETL	Direxion Daily Retail Bull 3X Shares	3x	$58.86	530 505
UTSL	Direxion Daily Utilities Bull 3X Shares	3x	$48.28	193 119
TYD	Direxion Daily 7-10 Year Treasury Bull 3x Shares	3x	$42.81	43 684
DUSL	Direxion Daily Industrials Bull 3X Shares	3x	$40.64	19 53
KORU	Direxion MSCI Daily South Korea Bull 3X Shares	3x	$40.61	443 984
UMDD	ProSharesUltraPro MidCap400	3x	$35.68	14 397
QTJA	Innovator Growth Accelerated Plus ETF – January	3x	$28.03	12 938
TPOR	Direxion Daily Transportation Bull 3X Shares	3x	$27.51	17 462
WANT	Direxion Daily Consumer Discretionary Bull 3X Shares	3x	$26.48	34 449
EURL	Direxion Daily FTSE Europe Bull 3x Shares	3x	$25.42	13 581
XTJA	Innovator U.S. Equity Accelerated Plus ETF – January	3x	$20.49	8 844
XTOC	Innovator U.S. Equity Accelerated Plus ETF – October	3x	$18.68	4 04
MEXX	Direxion Daily MSCI Mexico Bull 3X Shares	3x	$16.29	13 014
PILL	Direxion Daily Pharmaceutical & Medical Bull 3X Shares	3x	$13.64	83 376
QTOC	Innovator Growth Accelerated Plus ETF – October	3x	$12.66	4 19
FLYU	MicroSectors Travel 3x Leveraged ETN	3x	$8.57	2 014
CARU	MAX Auto Industry 3X Leveraged ETN	3x	$3.40	414

Source: Own elaboration, based on etfdb.com (2024), available on https://etfdb.com/etfs/leveraged/#etfs&sort_name=three_month_average_volume&sort_order=desc&page=6

The Table 4 shows the prevalence of 3x leveraged ETFs, such as biggest LETFs in the way of managed assets like TQQQ, SOXL, and TMF, which have more risk associated with leverage but may potentially yield greater gains.

The list offers tailored exposure to a variety of industries, including technology industry, finance, biotech, commodities, and others. The list also provides geographical diversity and areas like China (YINN) also demonstrate investor interest in worldwide diversity. LETFs, like FNGU and NRGU, provide exposure to niche indices.

Table 5: List of 2x Leveraged ETFs (April 2024)

Symbol	ETF Name	Leveraged	Total Assets ($MM)	Avg Volume
QLD	ProShares Ultra QQQ	2x	$6,207.89	3 552 654
SSO	ProShares Ultra S&P 500	2x	$5,105.08	3 464 840
BITX	2x Bitcoin Strategy ETF	2x	$1,476.03	3 213 141
USD	ProShares Ultra Semiconductors	2x	$856.08	370 568
UYG	ProShares Ultra Financials	2x	$711.42	39 576
ROM	ProShares Ultra Technology	2x	$686.66	70 525

NUGT	Direxion Daily Gold Miners Index Bull 2x Shares	2x	$639.23	2 843 251
UCO	ProShares Ultra Bloomberg Crude Oil	2x	$585.80	2 235 003
GUSH	Direxion Daily S&P Oil & Gas Exp. & Prod. Bull 2X Shares	2x	$545.06	871 075
CONL	GraniteShares 2x Long COIN Daily ETF	2x	$473.20	1 196 298
ERX	Direxion Daily Energy Bull 2X Shares	2x	$436.40	607 554
AGQ	ProShares Ultra Silver	2x	$430.36	1 298 837
DDM	ProShares Ultra Dow30	2x	$408.69	320 06
XDEC	FT Vest U.S. Equity Enhance & Moderate Buffer ETF – December	2x	$385.28	59 192
CWEB	Direxion Daily CSI China Internet Index Bull 2x Shares	2x	$354.63	758 546
JNUG	Direxion Daily Junior Gold Miners Index Bull 2x Shares	2x	$323.76	1 723 844
FNGO	MicroSectors FANG+ Index 2X Leveraged ETNs	2x	$290.96	44 629
UWM	ProShares Ultra Russell2000	2x	$238.28	949 986
UGL	ProShares Ultra Gold	2x	$219.60	143 127
FBGX	UBS AG FI Enhanced Large Cap Growth ETN	2x	$202.35	273
MVV	ProShares Ultra MidCap400	2x	$149.91	16 335
SPUU	Direxion Daily S&P 500 Bull 2x Shares	2x	$149.40	19 19
XSEP	FT Vest U.S. Equity Enhance & Moderate Buffer ETF – September	2x	$132.36	13 749
DIG	ProShares Ultra Energy	2x	$121.36	79 311
MSOX	AdvisorShares MSOS 2x Daily ETF	2x	$106.58	1 614 116
XBJA	Innovator U.S. Equity Accelerated 9 Buffer ETF – January	2x	$100.10	34 829
BIB	ProShares Ultra Nasdaq Biotechnology	2x	$98.23	42 502
BRZU	Direxion Daily MSCI Brazil Bull 2X Shares	2x	$97.16	28 006
RXL	ProShares Ultra Health Care	2x	$94.82	3 122
DGP	DB Gold Double Long Exchange Traded Notes	2x	$94.11	6 619
INDL	Direxion Daily MSCI India Bull 2X Shares	2x	$86.41	26 995
UVIX	2x Long VIX Futures ETF	2x	$75.75	5 334 397
XBOC	Innovator U.S. Equity Accelerated 9 Buffer ETF – October	2x	$72.44	15 052
CHAU	Direxion Daily CSI 300 China A Share Bull 2x Shares	2x	$68.76	160 675
URE	ProShares Ultra Real Estate	2x	$62.61	5 714
TARK	AXS 2X Innovation ETF	2x	$58.07	104 078
UYM	ProShares Ultra Materials	2x	$46.19	7 456
FEDL	ETRACS 2x Leveraged IFED Invest with the Fed TR Index ETN	2x	$44.10	257
FNGG	Direxion Daily NYSE FANG+ Bull 2X Shares	2x	$43.89	15 363
QULL	ETRACS 2x Leveraged MSCI US Quality Factor TR ETN	2x	$41.80	383
UBOT	Direxion Daily Robotics, Artificial Intelligence & Automation Index Bull 2X Shares	2x	$40.85	54 06
EUO	ProSharesUltraShort Euro	2x	$39.05	19 44
SCDL	ETRACS 2x Leveraged U.S. Dividend Factor TR ETN	2x	$36.85	154
SMHB	ETRACS 2xMonthly Pay Leveraged US Small Cap High Dividend ETN Series B	2x	$36.49	41 932
IWDL	ETRACS 2x Leveraged US Value Factor TR ETN	2x	$36.18	310
IWFL	ETRACS 2x Leveraged US Growth Factor TR ETN	2x	$35.47	506
USML	ETRACS 2x Leveraged MSCI US Minimum Volatility Factor TR ETN	2x	$34.50	476
SAA	ProShares Ultra SmallCap600	2x	$34.17	10 83
YCS	ProSharesUltraShort Yen	2x	$32.79	14 683

ESUS	ETRACS 2x Leveraged MSCI US ESG Focus TR ETN	2x	$27.50	152
MTUL	ETRACS 2x Leveraged MSCI US Momentum Factor TR ETN	2x	$25.65	662
UXI	ProShares Ultra Industrials	2x	$24.57	6 987
IWML	ETRACS 2x Leveraged US Size Factor TR ETN	2x	$19.05	1 24
UCC	ProShares Ultra Consumer Discretionary	2x	$18.18	5 059
EET	ProShares Ultra MSCI Emerging Markets	2x	$15.47	4 106
UST	ProShares Ultra 7-10 Year Treasury	2x	$14.20	10 578
EZJ	ProShares Ultra MSCI Japan	2x	$12.78	12 024
HDLB	ETRACS Monthly Pay 2xLeveraged US High Dividend Low Volatility ETN Series B	2x	$12.19	2 667
OOTO	Direxion Daily Travel & Vacation Bull 2X Shares	2x	$11.96	9 522
UPW	ProShares Ultra Utilities	2x	$11.36	2 259
PFFL	ETRACS 2xMonthly Pay Leveraged Preferred Stock Index ETN	2x	$10.17	4 276
EFO	ProShares Ultra MSCI EAFE	2x	$10.01	3 589
XPP	ProShares Ultra FTSE China 50	2x	$9.10	18 9
XDJA	Innovator U.S. Equity Accelerated ETF – January	2x	$7.19	2 276
XDOC	Innovator U.S. Equity Accelerated ETF – October	2x	$7.11	2 452
UGE	ProShares Ultra Consumer Staples	2x	$7.10	5 998
UJB	ProShares Ultra High Yield	2x	$6.82	21 316
CLDL	Direxion Daily Cloud Computing Bull 2X Shares	2x	$6.53	29 175
SHNY	MicroSectors Gold 3X Leveraged ETNs	2x	$5.54	5 789
UPV	ProShares Ultra FTSE Europe	2x	$4.72	256
LTL	ProShares Ultra Communication Services	2x	$4.47	6 781
UBR	ProShares Ultra MSCI Brazil Capped	2x	$4.40	1 2
UCYB	ProShares Ultra Nasdaq Cybersecurity ETF	2x	$4.13	4 195
EVAV	Direxion Daily Electric and Autonomous Vehicles Bull 2X Shares	2x	$3.75	11 871
SKYU	ProShares Ultra Nasdaq Cloud Computing ETF	2x	$3.63	2 59
KLNE	Direxion Daily Global Clean Energy Bull 2X Shares ETF	2x	$3.62	13 371

Source: Own elaboration, based on etfdb.com (2024), available on https://etfdb.com/etfs/leveraged/#etfs&sort_name=three_month_average_volume&sort_order=desc&page=6

The Table 5, taken as a whole, shows the depth and variety of 2x leveraged ETF options, which appeal to investors looking for higher returns and deliberate portfolio allocations across different industries and themes. The information on average trading volumes, total assets under management (AUM), their leverage ratios, and ETF names and symbols is included in the table. Remarkably, 2x leveraged exchange-traded funds aspire to offer twice the daily return of the corresponding underlying indices, drawing in investors looking for increased exposure to industries or asset classes. Significant AUM and trading volumes are demanded by symbols like QLD, SSO, and BITX, indicating investor enthusiasm in taking advantage of market opportunities.

These LETFs offer specialized exposure to market niches and cover a range of sectors, including technology (ROM, CWEB), finance (UYG, FNGO), commodities (UCO, AGQ), and emerging markets (BRZU, INDL). Leveraged ETFs that offer exposure to precious metals, such as USD, UGL, and DGP, also satisfy investors' need for inflation hedges and alternative assets. Additionally, the table features cutting-edge ETF structures like leveraged ETNs (CONL, FNGG) and thematic ETFs that concentrate on popular themes like cloud computing (SHNY, SKYU), robotics (UBOT, EVAV), and clean energy (KLNE).

Table 6: List of Other Leveraged ETFs (April 2024)

Symbol	ETF Name	Leveraged	Total Assets ($MM)	Avg Volume
NVDL	GraniteShares 2x Long NVDA Daily ETF	1.5x	$1,941.19	11,289,756
TSLL	Direxion Daily TSLA Bull 2X Shares	1.5x	$753.71	15,355,254
UVXY	ProShares Ultra VIX Short-Term Futures ETF	1.5x	$281.73	26,720,064
MSFU	Direxion Daily MSFT Bull 2X Shares	1.5x	$80.15	220,03
AMZU	Direxion Daily AMZN Bull 2X Shares	1.5x	$76.02	229,395
FBL	GraniteShares 2x Long META Daily ETF	1.5x	$72.80	335,595
GGLL	Direxion Daily GOOGL Bull 2X Shares	1.5x	$64.23	194,759
MLPR	ETRACS Quarterly Pay 1.5x Leveraged Alerian MLP Index ETN	1.5x	$59.07	1,667
AAPU	Direxion Daily AAPL Bull 2X Shares	1.5x	$50.92	245,083
BDCX	ETRACS Quarterly Pay 1.5x Leveraged MarketVector BDC Liquid Index ETN	1.5x	$33.34	1,597
CEFD	ETRACS Monthly Pay 1.5x Leveraged Closed-End Fund Index ETN	1.5x	$19.56	2,77
MVRL	ETRACS Monthly Pay 1.5x Leveraged Mortgage REIT ETN	1.5x	$17.64	8,687
TSL	GraniteShares 1.25x Long Tesla Daily ETF	1.25x	$4.54	126,775

Source: Own elaboration, based on etfdb.com (2024), available on https://etfdb.com/etfs/leveraged/#etfs&sort_name=three_month_average_volume&sort_order=desc&page=6

The Table 6 lists the average trading volumes and total assets under management (AUM) of leveraged exchange-traded funds (ETFs) with leverage ratios lower than 2x.

The LETFs that have been featured give investors increased exposure to equities or indexes with the goal of generating returns that are 1.5x the daily performance of the underlying assets. Leveraged ETFs following well-known stocks like Nvidia, Tesla, and VIX short-term futures are represented by symbols like NVDL, TSLL, and UVXY, respectively.

Moreover, other leveraged ETFs targeting tech giants like Microsoft (MSFU), Amazon (AMZU), and Google (GGLL) are also included in the table to cater to investors looking for increased exposure to industries or businesses.

Additionally, the Table no. 6 includes leveraged exchange-traded notes (ETNs) that track specialist indexes, like MLPs (MLPR), BDCs (BDCX), closed-end funds (CEFD), and mortgage REITs (MVRL), giving investors focused exposure to market niches.

GraniteShares provides a unique ETF (TSL) that provides marginally increased exposure to Tesla (TSLA) stock, with a leverage ratio of 1.25x.

To sum up, data from Investopedia.com[6] reveals that the most traded Leveraged ETF, based on three-month average daily trading volume, is the ProSharesUltraPro QQQ (TQQQ), with a three-month average daily volume of 72 147 056 and assets under management of more than $22 billion. This LETF provides 3x daily long exposure to the tech-heavy Nasdaq-100 Index.

Table 7: List of AUM Leaderboard (April 2024)

AUM Rank	Leveraged	Assets Under Management ($MM)	# of ETFs
1	Leveraged Equities	$84,466.21	114
2	Leveraged Bonds	$5,152.37	5
3	Leveraged Commodity	$1,877.87	7
4	Leveraged Currency	$1,581.56	5
5	Leveraged Volatilities	$344.90	2
6	Leveraged Real Estate	$149.92	3
7	Leveraged Multi-Assets	$19.80	1
8	Leveraged Preferred Stocks	$10.43	1

Source: Own elaboration, based on etfdb.com (2024), available on https://etfdb.com/etfs/leveraged/#etfs&sort_name=three_month_average_volume&sort_order=desc&page=6

In the Table 7, we provide the summary of the assets under management (AUM) rankings for the several classes of leveraged exchange-traded funds. With an AUM of over $84billions spread across 114 ETFs, leveraged equities is by far the largest category, reflecting strong investor demand in further exposure to the equity markets. Leveraged

[6] Investopedia.com [electronic source]. online. Available on: https://www.investopedia.com/articles/investing/020816/top-10-most-traded-leveraged-etfs-uvxy-sds.asp

bonds, with more than $5 billion AUM spread across 5 ETFs, come next among leveraged equities, attracting investors looking for leveraged exposure to fixed income instruments. The remaining categories, which each represent distinct market sectors with various AUM levels, are leveraged commodity, currency, volatilities, real estate, multi-assets, and preferred stocks. The data highlights the wide array of leveraged exchange-traded funds (ETFs) that investors can choose from, spanning many asset classes and market niches.

3.2. Pricing and volatility of LETFs

The chapter characterizes the fundamental properties of a pricing of LETF, which is key concept for understanding how LETFs generates profits. Although the factors listed below can be applied to additional types of ETFs, the thesis is focused on the LETFs.

An ETF is an investing instrument, and the so-called investment triangle is a crucial component that needs to be considered when assessing the worth or performance of a particular fund. The fundamental factors that all investors must consider are return, risk, and liquidity. This chapter will outline these qualities and further fundamental factors that determine the standing of LETFs and their pricing efficiency. The practical part of the thesis will then use the information from this chapter.

3.2.1. Closing price, premium and discount

The closing price is crucial for the computations since the LETFs are traded on the stock exchange all day long. The price at which the ETF (or LETF) closes at the conclusion of the trading day because of exchange-moving factors includes supply and demand for the ETF, the LETF's creation, dividend payments, fees, and other considerations. The premium or discount of the ETF can be computed using the difference between the closing price and the NAV. If the share price at closing exceeds the NAV, trading at a premium is taken into consideration. Therefore, the discount occurs in the opposite scenario, where the share's closing price is lower than the NAV. Thus, an investor can obtain information about how the LETF is trading. (Hill et. al, 2015).

LETF closing prices, or NAV, is one of several input variables that can be used to analyze the performance of LETFs. Since closing prices and NAVs are readily available to the public, there is no need to measure them independently. Certain indicators, such as the deviation of the closing prices of LETFs from the NAV, can be computed using the closing price and the NAV to measure how much the closing price deviates from the NAV.

3.2.2. Returns of Leveraged ETFs

The LETF closing prices can be used to compute the yield. Nonetheless, funds frequently operate using NAV, and many authors measure the fund's return using NAV. Authors use NAV more frequently because it gives an image of the internal LETF. Veselá (2019) explains following computation technique:

(1)

$$r^i = \frac{NAV_t^i - NAV_{t-1}^i + D^t}{NAV_{t-1}^i} * 100$$

NAV_t^i- net asset value of the i-th LETF in period t

NAV_{t-1}^i- net asset value of the i-th LETF in period t - 1

D^t - dividend paid during the period under consideration.

This approach of Veselá can be transformed into the following form used by Charupat and Miu (2011):

(2)

$$r^i = \frac{NAV_t^i}{NAV_{t-1}^i} - 1$$

The yield is the percentage change in the value of the NAV from the previous day. This difference may be seen in dividends, where formula 2 states that no dividend is considered.

When the process is applied to the NAV values, the yield that is produced is a historical but real variable that represents the actual yield the LETF obtained during a certain reporting period in relation to the NAV value of that specific LETF. Since LETFs offer multiple returns on the underlying index or other benchmark, Formula 3 can also be applied in the manner shown below.

To find the necessary (hypothetical) return of the LETF, which should be ideal in relation to the LETF's definition, Formula 2 might be used as a step in between.

Given that the LETF is viewed as an ETF that aims to consistently produce a multiple of the daily return of the underlying asset, the For the sake of this thesis, we refer to this constant multiple for LETFs as $\beta\epsilon$ {-3,-2,-1, +2, +3} and for the needs of this master thesis $\beta\epsilon${+2, +3}.

Then:

(3)

$$r_{LETF\ hyp} = \beta\, r_i$$

Formula 3 illustrates the optimal return that the LETF ought to offer. Several factors will not lead to this return. Their average difference can be calculated by deducting the actual realized return from the desired (hypothetical optimal) return. As a result, we can determine the degree to which the actual return is different from the planned return (which is determined by the duplication of the LETF returns). Leung and Ward (2015) computed the distribution of average returns for exchange-traded funds (LETFs). They found out that this disparity grows with increasing time periods in their studies.

The following example shows what happens and how ETF and LETF will behave when we keep leverage constant in a condition of random movements:

Table 8: Test of constant leverage in conditions of random movements

Day	Underlying asset change	ETF	%change	+2x LETF	% change
0		100		100	
1	-3%	97	-3%	94	-6%
2	3%	99,91	3%	99,64	6%
3	-3%	96,91	-3%	93,66	-6%
4	3%	99,82	3%	99,28	6%
5	-3%	96,83	-3%	93,32	-6%
6	3%	99,73	3%	98,92	6%
7	-3%	96,74	-3%	92,99	-6%
8	3%	99,64	3%	98,57	6%
9	-3%	96,65	-3%	92,65	-6%
10	3%	99,55	3%	98,21	6%

Source: Own elaboration

We have provided the test to verify the fact that LETFs is not an effective financial instrument for longer holding period. From the results shown in Table 8, we see that ETF recorded a small loss of 0,45% after 10 days of holding and 2x LETF ended up with a loss of 1,79%, which is in absolute value bigger than 2x result of ETF performance (-0,45%). Except for day 1, every day can be observed to support this. On day three, for instance, the

ETF has a net loss of 3,09% and the LETF has a net loss of 6,34%. The difference in absolute terms is bigger than 6,18%, which is twice the original ETF's return.

The results of this test provided an important proof for us, to set effective trading period in the strategy, where we will use LETFs as a trading asset.

3.3. Rebalancing, compounding and holding period returns

To maintain the same leverage or short ratio at the start of each day as it did at the initial public offering, leveraged and inverse ETFs internally rebalance their long and short holdings at the conclusion of each day.

A straightforward five-day computation of how compounding and rebalancing affect the returns of leveraged and inverse ETFs is provided in Table no. 9. Over the course of five days, the daily returns add up to 0.01%.

Table 9: Computation of the impact of rebalancing and compounding on LETFs effective holding period.

	Index returns		Traditional ETFs and Cash or Margin Debt			Leveraged and Inverse ETFs	
	a)	b)	c)	d)	e)	f)	g)
Day	Daily return	Cumulative return	Unlevered ETF	1000$ Short ETF	3000$ Long ETF	1x -1ETF	3x LETF
			$ 1 000,00	$ 1 000,00	$ 1 000,00	$ 1 000,00	$ 1 000,00
1	23%	23,00%	$ 1 230,00	$ 770,00	$ 1 690,00	$ 770,00	$ 1 690,00
2	-20%	-1,60%	$ 984,00	$ 1 012,32	$ 918,88	$ 924,00	$ 676,00
3	20%	18,08%	$ 1 180,80	$ 816,97	$ 1 498,40	$ 739,20	$ 1 081,60
4	-23%	-9,08%	$ 909,22	$ 1 074,17	$ 591,91	$ 909,22	$ 335,30
5	10%	0,01%	$ 1 000,14	$ 999,85	$ 1 000,24	$ 818,29	$ 435,88

Source: Own elaboration

In the Table 9, we used various assets where we analyzed the impact of rebalancing and compounding on the effective holding period.

Unlevered ETF - is a typical ETF that seeks to track, devoid of leverage, the performance of an underlying asset or index. Its only goal is to replicate its benchmark's results. SPY is an example of an unlevered exchange-traded fund (ETF) that tracks the S&P 500 index.

1x -1ETF – this is an (1x) inverse ETF, which is designed to perform in the opposite direction of the underlying index it tracks. So, if the index loses value, the inverse

ETF will gain value. An example of inverse ETF is the ProShares Short S&P500 (SH), which seeks to deliver the inverse of the daily performance of the S&P 500.

3x LETF - A 3x Leveraged Exchange-Traded Fund (LETF) seeks to deliver three times the daily performance of the underlying index, both gains and losses. For instance, the WisdomTree S&P 500 3x Daily Leveraged (3USS) attempts to provide three times the daily performance of the S&P 500.

According to Column c), an $1000 investment was made in an unlevered ETF, on the first day of the week, and it is worth $1000,14 at the end of the period. Column d) illustrates the $1000 short that concluded the trading week worth $999,85. The comparable outcome of the investment $1000 into inverse ETF of the same underlying asset is shown in Column f) with the result of $818,29 after the same period.

Based on the results of the experiment that we obtained, we can say that the $1000 investment into inverse ETF will be less efficient than $1000 short of the same ETF.

The outcome of leveraging up a conventional ETF 3-to-1 throughout the same 5-day period is displayed in Column e) resulting into very small profit of $0,24. On the other hand, the result of 3x LETF after the same time period was - $564,12.

This simulation demonstrates the "constant leverage trap." The ETF's returns during a 5-day period were almost zero, along with the leverage and a short investment. On the other hand, an inverse ETF investment lost 18.2%, and a 3x leveraged ETF investment lost 56.4%. These significant losses are brought on by the daily compounding that these investments do, where long-term investors can experience big losses.

Wang et al. (2009) explains that the return on a leveraged ETF is based on the assumption:

(4)

$$(1 + R_T^{L-ETF}) = (1 + R_T^{Index})^x * e^{\frac{(x-x^2)\sigma^2 T}{2}}$$

where T is the time, that the investment is kept, σ is the index's volatility, and x is the leverage ratio. Across all market leveraged ETFs, the scalar term $e^{\frac{(x-x^2)\sigma^2 T}{2}}$ is positive and lower than one and goes more to 0 for longer holding periods. As a result, the return of

the leveraged ETF is determined by multiplying the underlying index's return by the leverage, times a multiplier that starts at 0 and increases in value until 1.

Therefore, the constant will be modest if the volatility is high or the holding period is longer, which might result in a lower return on the leveraged ETF than on the underlying index.

In the following assumption we investigated the behavior of each of chosen ETFs and LETFs following the same underlying asset within the various holding periods. Obtained outcomes are provided in following tables. The investigation was done on the period of 5 years. We were analyzing simple moving average of holding periods, starting by the period of 1 day, 2 days, until 15 days of holding that represents period of 3 trading weeks. Then 30 days, 90 days, and long-term period of 150 trading days in the time span from March 2019 to March 2024.

Table 10: Holding periods of SPY

Holding period	Mean	Median	Standard deviation	Sample Variance	Range	Maximum	Minimum	Count
1 day	0,06%	0,09%	1,34%	0,02%	21,37%	9,38%	-11,98%	1259
2 days	0,11%	0,23%	1,70%	0,03%	24,58%	10,64%	-13,93%	1258
3 days	0,17%	0,34%	2,12%	0,04%	30,51%	17,55%	-12,96%	1257
4 days	0,22%	0,44%	2,43%	0,06%	31,32%	14,11%	-17,21%	1256
5 days	0,28%	0,53%	2,68%	0,07%	35,36%	17,40%	-17,97%	1255
6 days	0,33%	0,61%	2,93%	0,09%	36,27%	15,52%	-20,75%	1254
7 days	0,39%	0,73%	3,09%	0,10%	33,72%	12,63%	-21,09%	1253
8 days	0,44%	0,83%	3,32%	0,11%	38,97%	15,20%	-23,77%	1252
9 days	0,49%	0,93%	3,48%	0,12%	37,88%	15,51%	-22,37%	1251
10 days	0,55%	1,01%	3,73%	0,14%	42,44%	19,05%	-23,39%	1250
11 days	0,61%	1,15%	3,92%	0,15%	43,59%	18,86%	-24,73%	1249
12 days	0,66%	1,34%	4,11%	0,17%	49,27%	22,91%	-26,36%	1248
13 days	0,71%	1,43%	4,31%	0,19%	53,21%	24,69%	-28,52%	1247
14 days	0,77%	1,54%	4,46%	0,20%	49,11%	23,43%	-25,68%	1246
15 days	0,82%	1,60%	4,64%	0,22%	54,80%	27,20%	-27,60%	1245
30 days	1,55%	2,70%	6,20%	0,38%	60,31%	27,06%	-33,25%	1231
90 days	4,26%	5,52%	9,49%	0,90%	73,27%	45,64%	-27,64%	1171
150 days	7,00%	8,49%	12,22%	1,49%	77,83%	54,35%	-23,47%	1111

Source: Own elaboration

Table 11: Holding periods of 3USL

Holding period	Mean	Median	Standard deviation	Sample Variance	Range	Maximum	Minimum	Count
1 day	0,14%	0,28%	3,55%	0,13%	51,45%	24,85%	-26,59%	1259
2 days	0,29%	0,45%	5,07%	0,26%	65,38%	36,90%	-28,48%	1258
3 days	0,44%	0,77%	6,23%	0,39%	82,54%	50,63%	-31,91%	1257
4 days	0,58%	1,06%	7,16%	0,51%	81,32%	40,42%	-40,90%	1256
5 days	0,73%	1,28%	7,98%	0,64%	99,22%	51,30%	-47,93%	1255
6 days	0,88%	1,62%	8,69%	0,76%	106,59%	57,32%	-49,26%	1254
7 days	1,02%	1,83%	9,22%	0,85%	91,29%	39,59%	-51,70%	1253
8 days	1,16%	2,26%	9,75%	0,95%	99,89%	44,08%	-55,82%	1252
9 days	1,30%	2,63%	10,31%	1,06%	108,64%	47,58%	-61,07%	1251
10 days	1,45%	2,99%	10,92%	1,19%	110,41%	49,87%	-60,54%	1250
11 days	1,60%	3,21%	11,49%	1,32%	131,91%	68,20%	-63,71%	1249
12 days	1,76%	3,28%	12,05%	1,45%	134,97%	66,95%	-68,02%	1248
13 days	1,91%	3,62%	12,64%	1,60%	149,18%	81,59%	-67,59%	1247
14 days	2,06%	4,08%	13,14%	1,73%	153,03%	86,19%	-66,83%	1246
15 days	2,21%	4,17%	13,56%	1,84%	142,83%	76,40%	-66,43%	1245
30 days	4,11%	7,39%	18,22%	3,32%	174,03%	97,82%	-76,20%	1231
90 days	10,37%	13,40%	31,38%	9,85%	236,08%	166,44%	-69,63%	1171
150 days	16,78%	19,08%	42,73%	18,26%	285,13%	220,50%	-64,62%	1111

Source: Own elaboration

In the tables 10 and 11 is shown the outcome of statistical analysis, of the performance of assets based on S&P 500 Index: SPY and 3USL.

Average return over the various holding periods was the key indicator in the analysis. Comparing the obtained results, data shows that ETFs had lower mean and median returns over the course of all holding periods. Based on this observation, the prediction that LETF should provide greater returns is confirmed in all analyzed holding periods.

Unless from the obtained data we can assume that the efficiency of LETF was lower than efficiency of simple ETF. When we multiply the average performance of simple ETF by the leverage (3), in every holding the performance of LETF is lower. We assume that the longer holding period of LETF was the lower efficiency it brought. The gap between the efficiency was constantly growing. In the holding period of 1 day, the gap was small (0,04%,) but as the holding period was longer, the gap was getting bigger up to 4,22% in the holding period of 150 days.

In addition, the range of returns as well as the highest and lowest values highlights the risk and volatility of 3USL compared to SPY. The maximum and minimum returns for

3USL were significantly greater than the returns of SPY in absolute terms, highlighting the higher vulnerability of LETFs to market fluctuations.

Table 12: Holding periods of DJI

Holding period	Mean	Median	Standard deviation	Sample Variance	Range	Maximum	Minimum	Count
1 day	0,04%	0,07%	1,31%	0,02%	24,29%	11,37%	-12,93%	1259
2 days	0,08%	0,16%	1,66%	0,03%	29,29%	14,03%	-15,26%	1258
3 days	0,12%	0,21%	2,10%	0,04%	35,59%	21,30%	-14,29%	1257
4 days	0,16%	0,28%	2,41%	0,06%	36,92%	17,62%	-19,30%	1256
5 days	0,20%	0,32%	2,66%	0,07%	38,93%	20,09%	-18,84%	1255
6 days	0,24%	0,38%	2,92%	0,09%	39,83%	17,89%	-21,95%	1254
7 days	0,28%	0,46%	3,08%	0,09%	37,02%	14,31%	-22,71%	1253
8 days	0,32%	0,53%	3,32%	0,11%	40,65%	15,18%	-25,48%	1252
9 days	0,36%	0,59%	3,48%	0,12%	40,84%	15,15%	-25,69%	1251
10 days	0,40%	0,64%	3,72%	0,14%	48,54%	21,99%	-26,55%	1250
11 days	0,44%	0,75%	3,90%	0,15%	49,97%	21,85%	-28,12%	1249
12 days	0,48%	0,82%	4,09%	0,17%	55,27%	26,04%	-29,22%	1248
13 days	0,52%	0,89%	4,28%	0,18%	58,95%	27,58%	-31,37%	1247
14 days	0,56%	1,01%	4,43%	0,20%	54,08%	25,81%	-28,26%	1246
15 days	0,61%	1,02%	4,61%	0,21%	59,19%	28,82%	-30,38%	1245
30 days	1,14%	1,92%	6,13%	0,38%	64,24%	27,74%	-36,50%	1231
90 days	3,05%	3,20%	9,06%	0,82%	75,61%	42,75%	-32,86%	1171
150 days	4,79%	4,47%	11,11%	1,23%	81,42%	52,56%	-28,86%	1111

Source: Own elaboration

Table 13: Holding periods of UDOW

Holding period	Mean	Median	Standard deviation	Sample Variance	Range	Maximum	Minimum	Count
1 day	0,74%	0,16%	13,12%	1,72%	167,77%	100,05%	-67,72%	1259
2 days	1,11%	0,47%	16,09%	2,59%	216,90%	152,49%	-64,41%	1258
3 days	1,18%	0,52%	16,12%	2,60%	189,66%	118,65%	-71,01%	1257
4 days	1,28%	0,66%	16,51%	2,73%	202,95%	132,50%	-70,45%	1256
5 days	1,46%	0,81%	17,66%	3,12%	273,40%	199,04%	-74,36%	1255
6 days	1,24%	0,88%	15,27%	2,33%	203,69%	142,51%	-61,18%	1254
7 days	1,37%	1,10%	15,97%	2,55%	249,59%	172,99%	-76,60%	1253
8 days	1,62%	1,32%	17,38%	3,02%	207,30%	138,34%	-68,96%	1252
9 days	1,82%	1,30%	18,53%	3,43%	221,45%	153,76%	-67,69%	1251
10 days	1,86%	1,36%	18,56%	3,45%	233,93%	170,34%	-63,59%	1250
11 days	1,91%	1,75%	18,33%	3,36%	237,36%	169,95%	-67,42%	1249
12 days	1,86%	1,95%	17,59%	3,09%	235,16%	167,34%	-67,81%	1248
13 days	2,08%	2,08%	18,62%	3,47%	250,60%	179,41%	-71,19%	1247
14 days	2,27%	2,47%	19,46%	3,79%	237,39%	166,77%	-70,62%	1246
15 days	2,44%	2,51%	20,24%	4,10%	271,88%	197,26%	-74,62%	1245
30 days	3,76%	4,21%	23,78%	5,65%	268,76%	182,93%	-85,83%	1231
90 days	7,19%	6,41%	34,46%	11,87%	324,88%	247,83%	-77,05%	1171
150 days	9,85%	6,58%	40,28%	16,23%	465,55%	393,18%	-72,37%	1111

Source: Own elaboration

In the tables 12 and 13 is shown the outcome of statistical analysis, of the performance of assets based on Dow Jones Industrial Average: DJI and UDOW.

Average return over the various holding periods is the key indicator in the analysis. Comparison of the results showed that simple ETFs had lower mean and median returns over the course of all holding periods, meaning that LETF provided higher total returns in all analyzed holding periods.

From the obtained results we assume that the efficiency of LETF was higher than efficiency of simple ETF in holding period of 30-days and lower. In the longer holding period (more than 30-days) the LETF was becoming more and more inefficient. In the holding period of 1 day, the efficiency of LETF was higher and gap was 0,62%, but as the holding period was longer, the gap is getting tighter, and in the holding period of 150 days, simple ETF was more efficient with the gap of 4,52%.

The range of returns as well as the highest and lowest values are more than 3 times higher for LETF and highlights the risk and volatility of UDOW. The maximum and minimum returns of UDOW were significantly greater than returns of DJI in absolute terms.

Table 14: Holding periods of QQQ

Holding period	Mean	Median	Standard deviation	Sample Variance	Range	Maximum	Minimum	Count
1 day	0,07%	0,13%	1,58%	0,02%	21,67%	9,35%	-12,32%	1259
2 days	0,14%	0,29%	2,05%	0,04%	23,06%	9,37%	-13,69%	1258
3 days	0,21%	0,42%	2,52%	0,06%	26,83%	13,66%	-13,17%	1257
4 days	0,28%	0,50%	2,88%	0,08%	30,60%	13,34%	-17,25%	1256
5 days	0,35%	0,65%	3,18%	0,10%	30,90%	13,31%	-17,59%	1255
6 days	0,43%	0,80%	3,49%	0,12%	35,64%	15,50%	-20,14%	1254
7 days	0,49%	0,96%	3,70%	0,14%	34,82%	13,84%	-20,99%	1253
8 days	0,56%	1,07%	3,97%	0,16%	39,16%	15,72%	-23,44%	1252
9 days	0,63%	1,26%	4,16%	0,17%	37,81%	17,32%	-20,49%	1251
10 days	0,70%	1,43%	4,44%	0,20%	38,98%	16,11%	-22,87%	1250
11 days	0,77%	1,64%	4,66%	0,22%	38,79%	17,52%	-21,27%	1249
12 days	0,84%	1,76%	4,89%	0,24%	41,65%	17,93%	-23,71%	1248
13 days	0,91%	1,87%	5,12%	0,26%	42,77%	18,85%	-23,92%	1247
14 days	0,98%	1,94%	5,32%	0,28%	44,21%	19,41%	-24,80%	1246
15 days	1,05%	2,06%	5,54%	0,31%	50,27%	24,12%	-26,14%	1245
30 days	1,99%	3,17%	7,55%	0,57%	56,28%	27,54%	-28,75%	1231
90 days	5,69%	6,43%	12,77%	1,63%	83,28%	55,06%	-28,22%	1171
150 days	9,67%	13,34%	17,55%	3,08%	102,02%	69,65%	-32,37%	1111

Source: Own elaboration

Table 15: Holding periods of TQQQ

Holding period	Mean	Median	Standard deviation	Sample Variance	Range	Maximum	Minimum	Count
1 day	0,24%	0,41%	4,72%	0,22%	61,47%	26,99%	-34,48%	1259
2 days	0,45%	0,84%	6,23%	0,39%	65,55%	28,81%	-36,74%	1258
3 days	0,66%	1,10%	7,61%	0,58%	78,57%	38,99%	-39,58%	1257
4 days	0,88%	1,31%	8,69%	0,75%	86,67%	39,30%	-47,37%	1256
5 days	1,10%	1,69%	9,61%	0,92%	82,44%	37,64%	-44,80%	1255
6 days	1,32%	2,07%	10,52%	1,11%	102,29%	50,75%	-51,54%	1254
7 days	1,52%	2,55%	11,16%	1,25%	99,82%	45,75%	-54,07%	1253
8 days	1,74%	2,74%	11,94%	1,43%	111,59%	53,49%	-58,10%	1252
9 days	1,94%	3,63%	12,58%	1,58%	110,39%	57,36%	-53,03%	1251
10 days	2,17%	4,11%	13,42%	1,80%	113,44%	55,99%	-57,45%	1250
11 days	2,39%	4,47%	14,11%	1,99%	116,11%	59,92%	-56,19%	1249
12 days	2,62%	4,65%	14,82%	2,19%	114,78%	54,75%	-60,04%	1248
13 days	2,85%	5,16%	15,52%	2,41%	113,54%	53,60%	-59,95%	1247
14 days	3,07%	5,56%	16,13%	2,60%	118,29%	58,88%	-59,41%	1246
15 days	3,30%	5,92%	16,81%	2,83%	140,63%	79,55%	-61,08%	1245
30 days	6,22%	8,84%	23,00%	5,29%	152,61%	84,72%	-67,89%	1231
90 days	17,66%	17,79%	44,02%	19,38%	285,51%	219,12%	-66,39%	1171
150 days	30,29%	35,69%	58,15%	33,81%	368,53%	295,70%	-72,82%	1111

Source: Own elaboration

In the tables 14 and 15 is presented the outcome of statistical analysis, of the performance of assets based on NASDAQ 100: QQQ and TQQQ.

The key indicator in the analysis was average return over the various holding periods. Comparison of the obtained results indicates that simple ETFs had lower average and median returns over the monitored holding periods, meaning that LETF delivered higher total returns in all holding periods.

From the obtained results we assume that the efficiency of TQQQ was higher in all holding periods. The gap of efficiency in the 1-day holding period was just 0,03%, and it was varying over the growing holding periods, but even after 150 days of holding the asset it was effective.

When we compared the range of returns and the highest and lowest values, it assumes the higher level of the risk and volatility of UDOW.

Trainor (2013) stated that investing in LETFs is not a wise long-term strategy. Leveraged ETF returns have an intrinsic predicted decay over time based on averages of volatility and market return. But in low volatility conditions, this decay is negligible and can even be overwhelmed by a strong trend.

We assume that the analyzed period was characterized by a strong downfall in April 2020 followed by huge rising trend.

Even though the longer holding period is not very efficient from investors point of view, more than 8% of investors in some leveraged ETFs have held the funds for longer than three months, indicating that some investors are, in fact, holding the least popular funds for extended periods of time (Guedj, Li, and McCann, 2010).

We can sum up those results obtained from experiment, proved that LETFs can be more effective than simple ETFs. However, it is important to mention that the efficiency is highly depending to the behavior of underlying asset and of the market behavior. Studies that have already been made, suggest that efficiency of LETFs is higher in shorter holding periods but with growing holding periods is getting lower. The result of our analysis corresponded with the ideas of studies, but we have to mention that the efficiency of LETFs is depending not just on holding period, but various factors play a role such as tracking error, market volatility or pricing of LETFs.

3.4. Statistical and Volatility analysis

As a first step in our analytical part, we have conducted the statistical analyses of the classic ETFs: SPY, DJI and QQQ for a given time period followed by volatility analyses. The total amount of data points that were observed is 1259 points across all indices, which represents trading period of 5 years.

Table 16: Statistical analyses of daily % price change of the SPY, DJI and QQQ

% Price Chg			
	SPY	**DJI**	**QQQ**
Mean	0,057%	0,041%	0,073%
Standard Error	0,04%	0,04%	0,04%
Median	0,09%	0,07%	0,13%
Standard Deviation	1,34%	1,31%	1,58%
Sample Variance	0,02%	0,02%	0,02%
Kurtosis	13,38	20,37	6,68
Skewness	-0,53	-0,57	-0,44
Range	21,37%	24,29%	21,67%
Maximum	9,38%	11,37%	9,35%
Minimum	-11,98%	-12,93%	-12,32%
Count	1259	1259	1259

Source: Own elaboration

Table 10 represents results of descriptive statistics analyze of daily percentage price changes of SPY, DJI and QQQ.

The daily percentage move for the DJI was 0.041% for the SPY 0.057%, and for the QQQ 0.073% on average. This suggests that relative to the other two indices, the QQQ experienced a little higher daily percentage change on average.

For all three indices, the standard error of the mean is 0.04%. This represents the accuracy with which the sample mean may be used to approximate the population mean. An estimate with a smaller standard error would be more accurate.

When the changes are sorted, the middle figure is known as the median daily percentage change, for SPY it is 0.09%, for DJI it is 0.07% and for QQQ 0.13%, respectively. The median indicates that QQQ had a stronger daily return midpoint and was less impacted by extreme values than the mean.

The standard deviation measures the amount of variability or dispersion from the mean. For DJI it was 1.31%, for the QQQ 1.58%, and the SPY has a standard deviation of 1.34%. This suggests higher volatility because QQQ's daily percentage movements were more widely dispersed from its mean.

Another way to quantify dispersion is the variance, which was equal to the square of the standard deviation. The variation of each of the three indices was 0.02%, or more precisely, a percentage squared.

Kurtosis measures the "tailedness" of the distribution. All distributions appeared to be leptokurtic (having large tails and a sharp peak in comparison to a normal distribution), with DJI exhibiting the most prominent tails, according to kurtosis values of 13.38 for SPY, 20.37 for DJI, and 6.68 for QQQ.

The highest daily percentage change in the sample was recorded for the DJI 11.37% for the SPY 9.38%, and for the QQQ 9.35%. The lowest recorded daily percentage change is the highest negative change. The minimum for the SPY is -11.98%, for the DJI -12.93%, and for the QQQ -12.32%.

The difference between the maximum and minimum is known as the range. The range of the DJI was 24.29% of QQQ 21.67%, and of SPY 21.37%. These values represent the overall distribution of data from the lowest to the largest daily percentage change.

Table 17: Volatility of SPY

VOLATILITY SPY		
	AVG VAR % 90d	**STD 90d**
MEAN	0,0638%	1,1740%
STD	0,1076%	0,6512%
MAX	0,8008%	3,3988%
MIN	-0,3209%	0,1965%

Source: Own elaboration

Table 18: Volatility of DJI

VOLATILITY DJI		
	AVG VAR % 90d	**STD 90d**
MEAN	0,0447%	1,0981%
STD	0,0948%	0,7131%
MAX	0,4146%	3,6578%
MIN	-0,4017%	0,0790%

Source: Own elaboration

Table 19: Volatility of QQQ

VOLATILITY QQQ		
	AVG VAR % 90d	**STD 90d**
MEAN	0,0810%	1,4588%
STD	0,1432%	0,6228%
MAX	1,1373%	3,3243%
MIN	-0,3457%	0,1649%

Source: Own elaboration

The tables 17, 18 and 19 show statistically measured 90-day standard deviation (STD 90d) and 90-day average variance percentage (AVG VAR % 90d) for the volatility of the SPY, DJI and QQQ over a given period of 5 years.

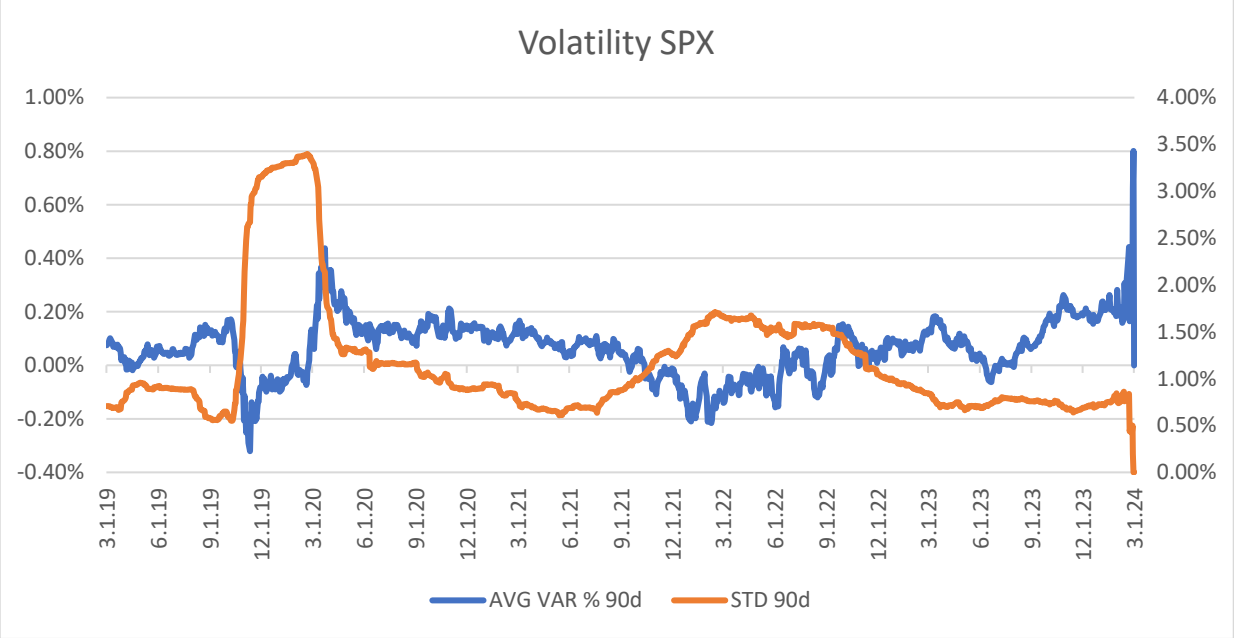
The QQQ had the highest mean average variance, followed by SPY and DJI. This figure shows the squared average of the index's return spread over a period of 90 days. Greater variation in returns throughout given period is indicated by a higher average variance.

The mean of 90-day standard deviation gives us a more logical way to quantify volatility because it is variance squared. The price changes of the index are more unpredictable when the standard deviation is bigger. The highest mean of 90-day standard deviation had QQQ followed by SPY and DJI, meaning that QQQ had highest dispersion of daily returns.

The highest 90-day standard deviation's standard deviation had DJI and conversely the lowest had QQQ. It shows the degree to which the volatility of the index deviates from its mean volatility. Bigger number indicates bigger volatility inconsistency.

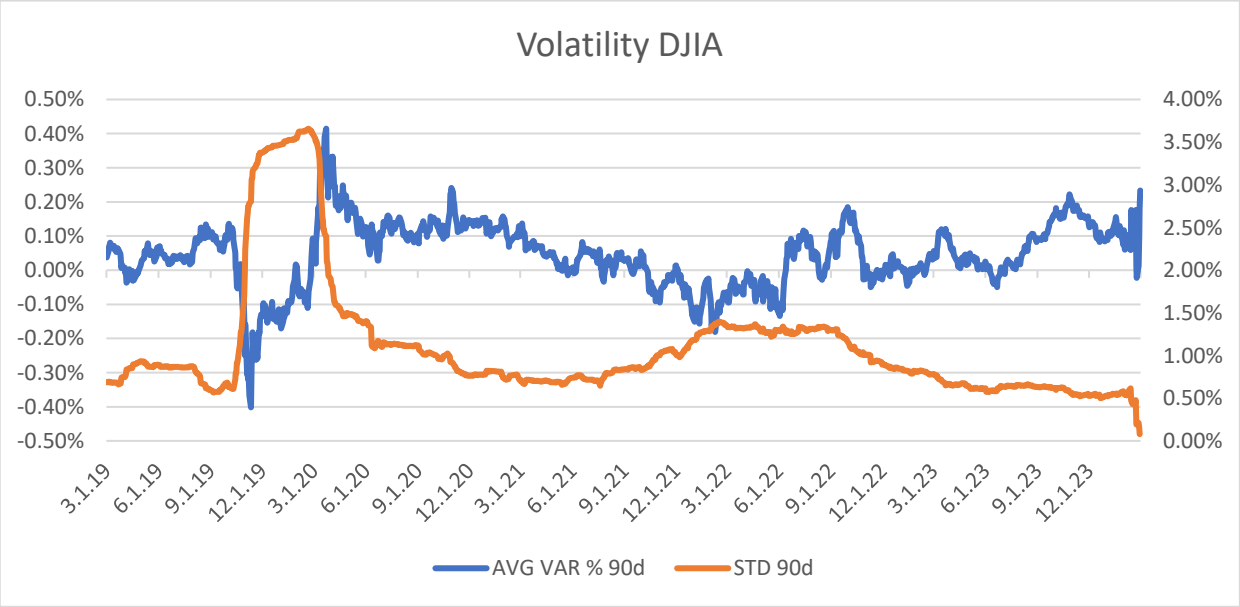
The index's least volatile 90-day period is represented by the minimal standard deviation which was the lowest for DJI.

Graph 8: Volatility of SPY (2019 – 2024)



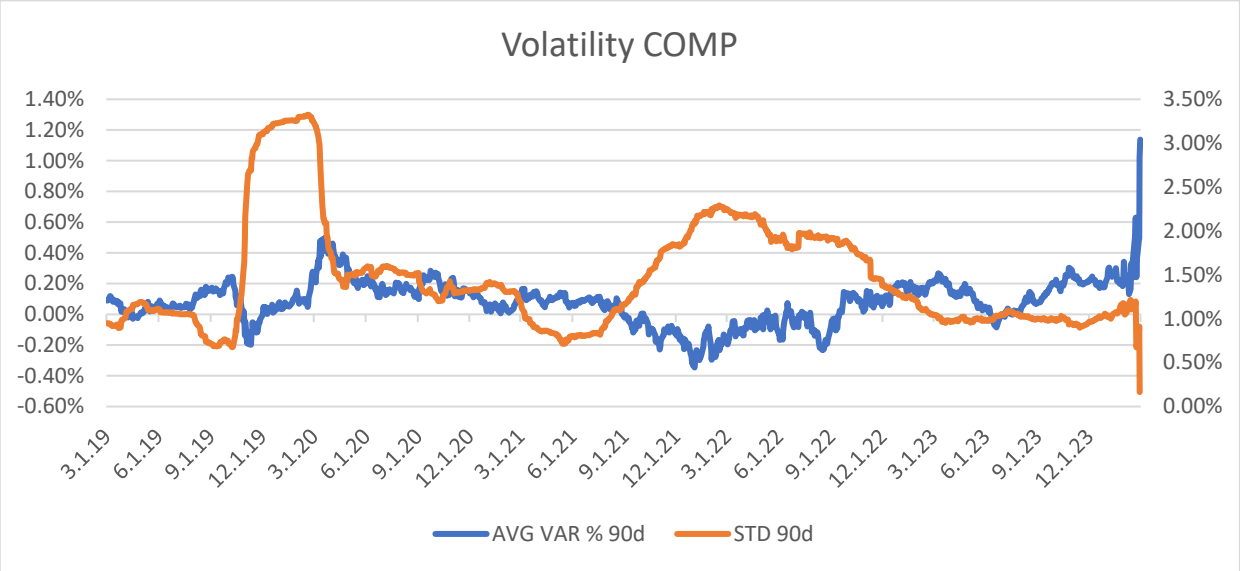
Source: Own elaboration

Graph 9: Volatility of DJI (2019 – 2024)



Source: Own elaboration

Graph 10: Volatility of QQQ (2019 – 2024)



Source: Own elaboration

To sum up, based on the 90-days statistically measured data, the ETF of NASDAQ Composite Index (QQQ) had the highest level of volatility, with more significant daily percentage changes and wider dispersion of returns compared to the DJI and SPY. The DJI was the least volatile in terms of daily percentage changes but experienced significant volatility on certain days. The ETF on S&P 500 Index (SPY) exhibited volatility levels between the DJI and QQQ, indicating it was moderately volatile relative to the other indexes.

3.5. Correlation and Beta Computation

This chapter presents comprehensive statistical analysis of correlation and Beta computation between chosen ETFs. We have employed regression analysis to investigate the interrelationships among the indexes and to understand how the dependent variable changes when the independent variables is varying.

Through this analysis, we aimed to discern the degree of correlation and association between the indexes and measure systematic risk. The results of the analysis were crucial for created investment strategy and for understanding the dynamics of the indexes.

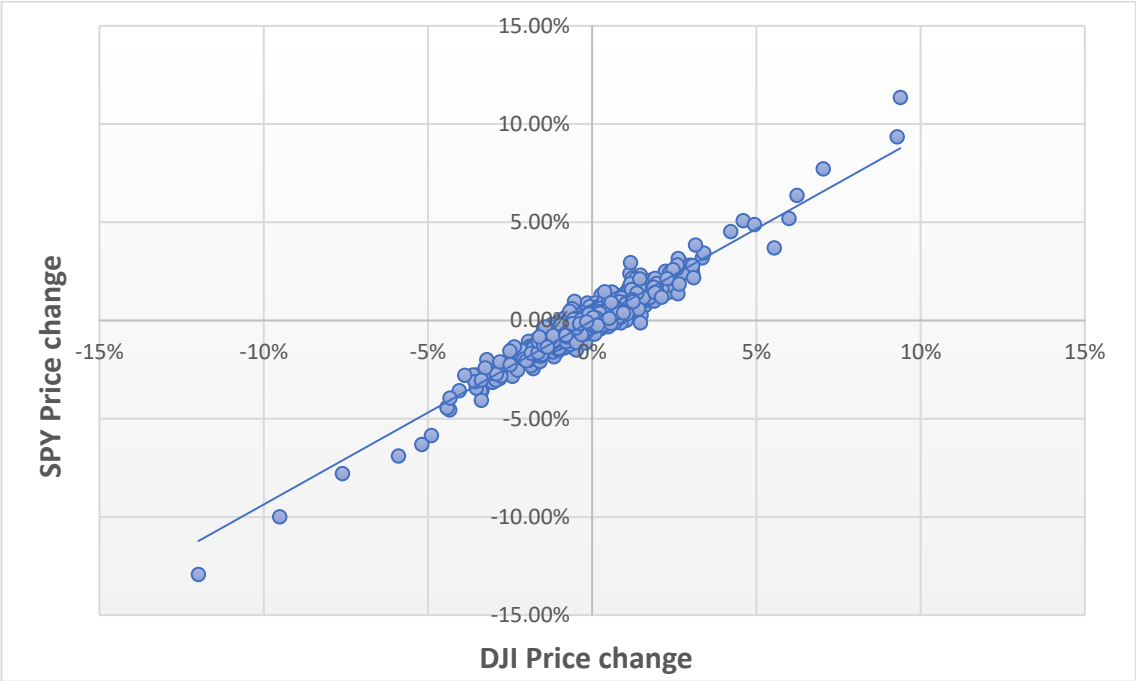
Table 20: Results of Correlation and Beta analysis

	SPY/DJI	SPY/QQQ	DJI/QQQ
Beta	0,936	1,1185	1,0118
Std. Error	0,0079	0,0107	0,0186
R2	0,917	0,897	0,7013
F	13783,2845	10939,5055	2949,5252
SS Reg	0,1974	0,2819	0,2204
Alpha	-0,0001	0,0001	0,0003
Std Er Y	0,0038	0,0051	0,0086
#Free	1256	1256	1256
SS Res	0,0179	0,0324	0,0938
Correlation	0,9576	0,9471	0,8375

Source: Own elaboration

Table 20 provides the overview of the results we obtained. The investigation included computation of Beta (β), which measured the relative volatility or systematic risk between the couple of ETFs.

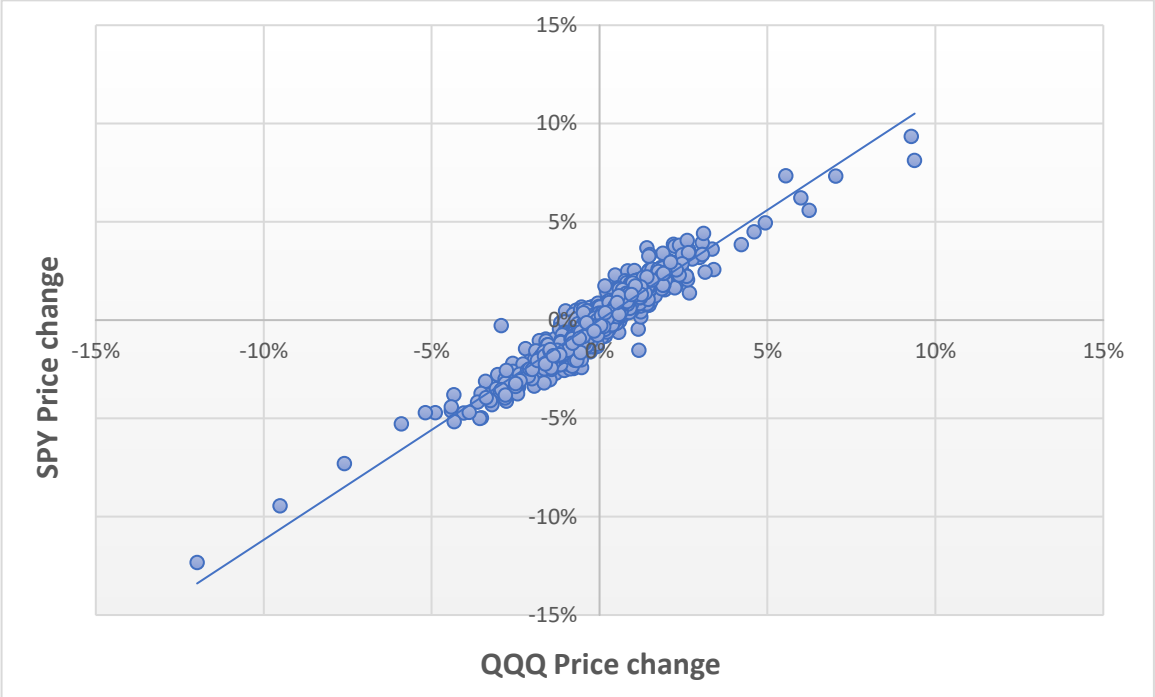
Graph 11: Classic Beta[7] of SPY/DJI



Source: Own elaboration

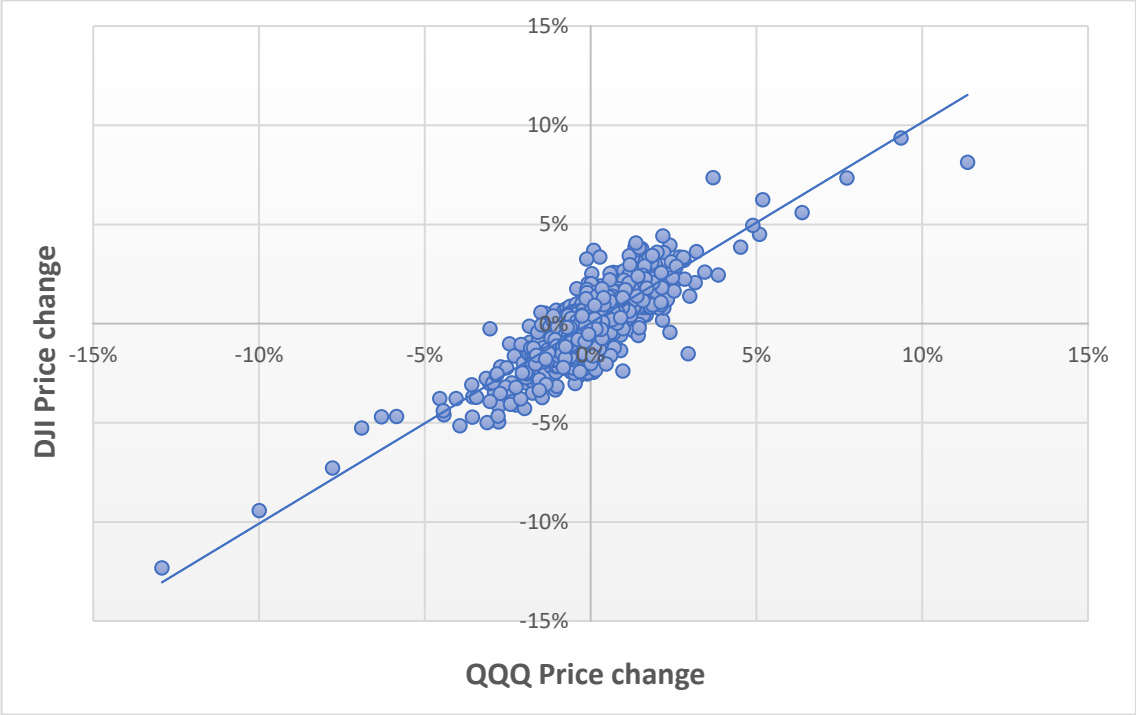
[7]Classic Beta of SPY/DJI constructed from daily prices for time span 5 years

Graph 12: Classic Beta[8] of SPY/QQQ



Source: Own elaboration

Graph 13: Classic Beta[9] of DJI/QQQ



Source: Own elaboration

8Classic Beta of SPY/QQQ constructed from daily prices for time span 5 years

9 Classic Beta of DJI/COMP constructed from daily prices for time span 5 years

Results that are shown in table 20 and graphs 12,13 and 14 suggests that for the movement of 1% of DJI, the SPY is expected to move 0.936% in the same direction, however SPY is more sensitive to the movements of the QQQ. For 1% change in the QQQ price, SPY is expected to change by approximately 1.1185%. On the other hand, DJI and QQQ have about the same level of volatility, with beta just over 1.

The Standard Error, indicates the precision of the regression estimate, meaning that low standard error in pair SPY/DJI suggests that the SPY's returns are predicted quite accurately. Higher standard errors in pairs SPY/QQQ and DJI/QQQ implied that precision of price prediction of SPY and DJI returns based on QQQ is lower.

R-squared (R^2) is reflecting the proportion of variance explained by model. High R^2 of the SPY/DJI indicated that DJI is very good predictor of the SPY's behavior.
also, R^2 of SPY/QQQ suggested strong relationship between indexes, however DJI/QQQ R^2 is significantly lower, meaning that QQQ was not explaining that much behavior of DJI, as it did for SPY.

Results of F-statistic explains the model's overall significance. SPY/DJI F-statistic is very large, suggesting that the model was highly significant. On the other hand, lower result for QQQ/DJI is still large number and good result, indicating significance.

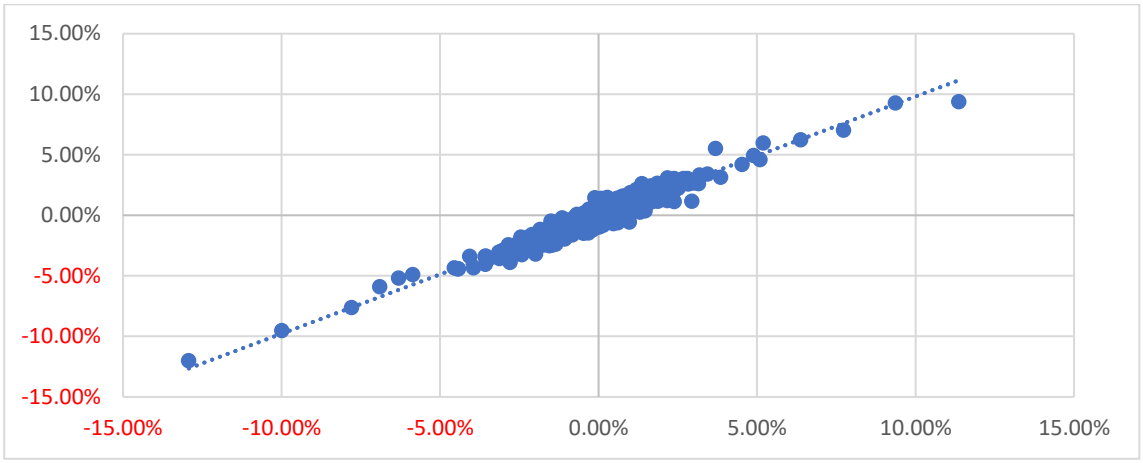
Sum of Squares due to Regression (SS Reg) and Residual (SS Res) are measuring the explained and unexplained variance respectively. SS Regression of SPY/DJI indicated good strength of model, where independent variable (DJI) was explaining depending variable (SPY) well. The other two pairs had higher SS Regression values that suggested more variability explained by the models. Lower values in SS Residuals indicates better fit. Lowest value of SPY/DJI reflected the portion of how total variance is explained by model.

Alpha (α) is representing the expected value when the independent variables are 0, meaning almost zero flat return for SPY when the DJI was not moving. The alpha results of SPY/QQQ and DJI/QQQ were positive but very small and showed negligible flat returns for both indexes when QQQ was not moving.

The degrees of freedom in all 3 cases were 1256, which is large enough to provide good statistical analysis.

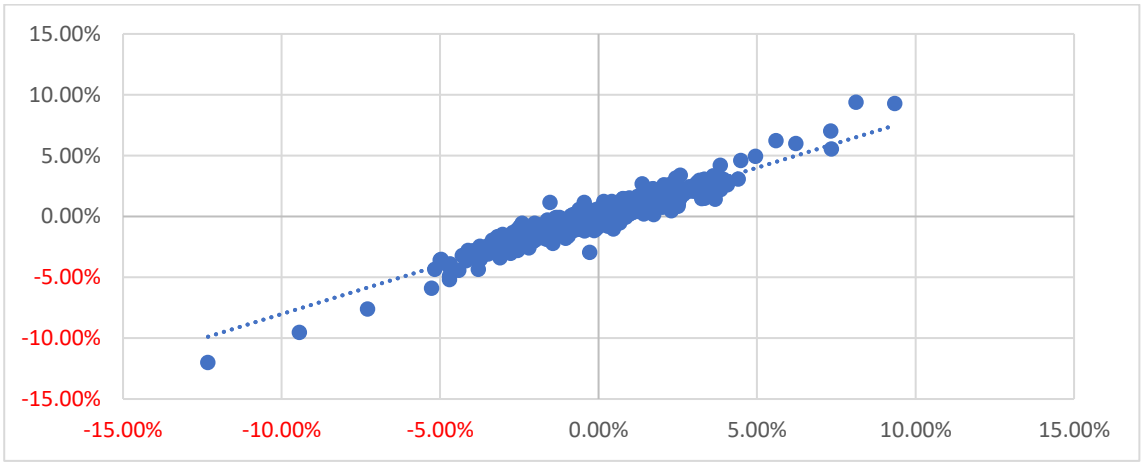
The last and the most important result we obtained is the Correlation between indexes in the pairs, quantifying the strength of linear relationship between indexes.

Graph 14: Correlation between SPY/DJI



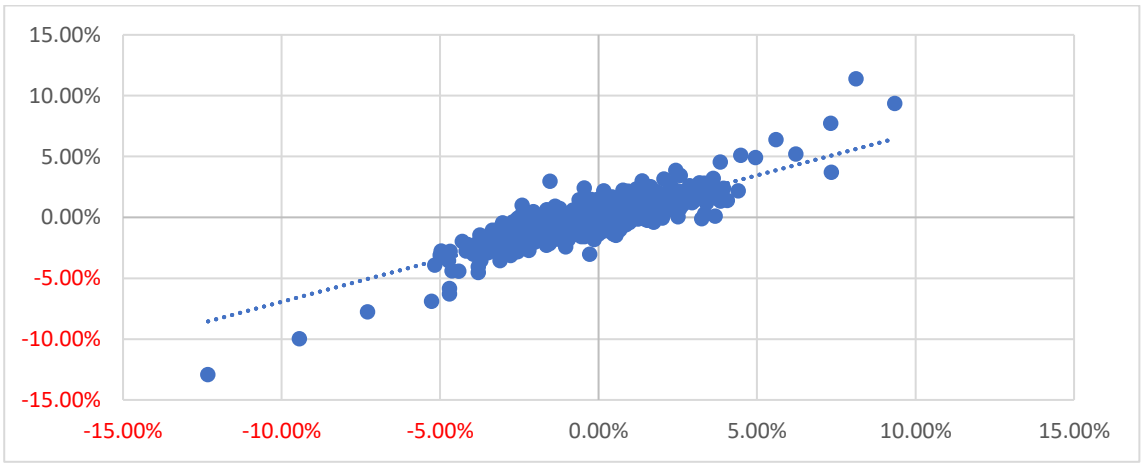
Source: Own elaboration

Graph 15: Correlation between SPY/QQQ



Source: Own elaboration

Graph 16: Correlation between SPY/QQQ



Source: Own elaboration

High correlation results for SPY/DJI and SPY/QQQ indicated that their movements were very coordinated, On the other hand DJI/QQQ correlation was smaller, which implied more independent movement of indexes.

The results of the statistical measures, we obtained suggested very strong relationship between S&P 500 ETF and other two indexes, with the relationship slightly stronger between SPY and DJI than the one between SPY and QQQ.

The movements of SPY were more closely mirrored by those of the DJI than by movements of the QQQ, as reflected higher R^2 and correlation, and the lower standard error and SS Residual.

To sum up, we assume that there are similar relationships between Leveraged ETFs as well. High correlation between underlying assets is main and key factor that must be satisfied for our investment strategy to be successful.

3.6. Investment strategy

This subchapter focuses on the statistical arbitrage strategy, its explanation, utilization, and application as an investment strategy using traditional ETFs and Leveraged ETFs.

3.6.1. Used ETFs

For the investment strategies we used indexes shown in the Tables 21 and 22. For the first strategy we used classic ETFs listed in Table 21 and for the second investment strategy we used Leveraged ETFs listed in Table 22.

Table 21: ETFs used in Investment Strategy

Ticker	Name of Index
SPY	S&P 500 Index
DJI	Dow Jones Industrial Average
QQQ	NASDAQ Composite Index

Source: Own elaboration

Table 22: Leveraged ETFs used in Investment strategy

Ticker	Name of Index
3USL	S&P 500 3X Daily Leveraged
3USS	S&P 500 3X Daily Short
UDOW	UltraPro Dow30 3x Shares
SDOW	UltraPro Short Dow30 -3x Shares
TQQQ	UltraPro QQQ 3x Shares
SQQQ	UltraPro Short QQQ -3x Shares

Source: Own elaboration

3.6.2. Arbitrage Pair Trading

Arbitrage and pair trading are two strategies that traders use to exploit market inefficiencies, but they can be combined in one strategy called “pairs trading arbitrage”. Before we go into the details of this combined strategy, let us break down the basics.

In order to profit from a price differential, arbitrage entails simultaneously buying and selling an asset or similar assets. When arbitrage is used properly, it is usually a risk-free approach since the transactions are made simultaneously to take advantage of the price difference. An occurrence of arbitrage is rare, pure arbitrage is often only feasible for short periods of time because high-frequency trading algorithms swiftly eradicate price discrepancies.

As a market-neutral tactic, pair trading entails placing opposing positions in two strongly linked assets. When their prices diverge, an investor will bet that the "spread" between the two will ultimately converge by shorting the outperforming asset and going long on the underperforming one.

This technique is predicated on the relative prices returning to a historical mean, rather than the direction of market moves.

The goal of pairs trading arbitrage is to take advantage of the price discrepancies between two linked assets. Pairs trading arbitrage is not risk-free, in contrast to pure arbitrage, as it depends on the possibility that the spread between the two assets will return to its historical mean, which may or may not occur.

We have chosen ETFs and LETFs that are based on the same underlying indexes. By performing analysis explained in chapter 3.4 and 3.5 we were assured that the pairs had

stable, long-term relationship and high correlation as a prediction that pairs are suitable for created strategy.

Firstly, we have transformed prices of the ETFs into their natural logarithmic form following by the computation of the Slope and Intercept of each pair.

Table 23: Slope and Intercept of Pairs

	SPY/DJI	SPY/QQQ	DJI/QQQ
Slope	0,74052	1,22930	1,52732
Intercept	4,24425	-0,77146	-6,44429

Source: Own elaboration

For every unit change in the independent variable, the slope shows the rate of change in the dependent variable (price ratio). When the independent variable is zero, the value of the dependent variable is called the intercept.

Secondly, we computed alpha spread and average and standard deviation of each pair.

Table 24: Average and Standard deviation of Alpha Spread

	SPY/DJI	SPY/QQQ	DJI/QQQ
Av. Alpha Spread	4,244247122	-0,771463676	-6,444287327
Std Alpha Spread	0,026992173	0,06581778	0,09805713

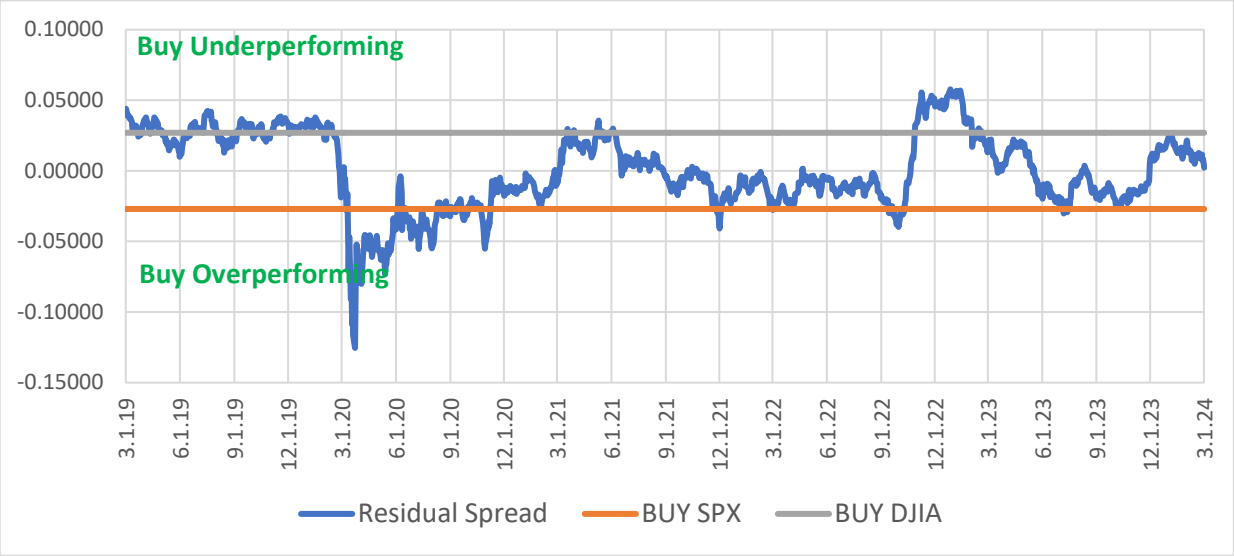
Source: Own elaboration

Computed statistical analysis of the average alpha spreads and the accompanying standard deviations is shown in the table 24. The average alpha spread shows how much the first index had performed better on average, than the second index. The volatility of this outperformance was quantified by the alpha spread standard deviation.

As a next step despite to Invesopedia.com[10] we have computed Residual spread multiplying alpha spread of each day by average Alpha spread. Then we have proceeded with the computation of Standard deviation of residual spreads.

[10] Investopedia.com [electronic source]. online. Available on: https://www.investopedia.com/terms/r/residual-standard-deviation.asp

Graph 17: Residual spread of SPY/DJI



Source: Own elaboration

The difference between the observed spread and the expected spread, predicted by a model is the residual spread. This residual spread is what we are looking at when we are trading pairs to figure out when is the optimal time to enter in the market.

3.6.3. Strategy rules

To make the strategy more efficient, we set up certain rules for trade execution.
First strategy rules, using only classic ETFs:

1. If the residual spread is positive and higher than the value of first standard deviation of alpha spread, it suggests that we will execute sell (short) of overperforming asset and at the same time we will execute buy (long) of underperforming asset.

2. If the residual spread is negative and lower than the reversed value of first standard deviation of alpha spread, it suggests that we will execute buy (long) of overperforming asset and at the same time we will execute sell (short) of underperforming asset.

3. If the residual spread is in the area between the value of first standard deviation of alpha spread and reversed value of first standard deviation of alpha spread, we don't execute any trades.

4. Holding period of the trade is one full trading day, so we open the trade in the beginning of the day, and we close it in the end of the trading day.

Second strategy rules, using only Leveraged ETFs:

1. If the residual spread is positive and higher than the value of first standard deviation of alpha spread, it suggests that we will execute buy of 3x Short Leveraged ETF of overvalued asset and at the same time we will execute of buy 3x Long Leveraged ETF of undervalued asset.

2. If the residual spread is negative and lower than the reversed value of first standard deviation of alpha spread, it suggests that we will execute buy of 3x Long Leveraged ETF of overperforming asset and at the same time we will execute buy of 3x Short Leveraged ETF of underperforming asset.

3. If the residual spread is in the area between the value of first standard deviation of alpha spread and reversed value of first standard deviation of alpha spread, we don't execute any trades.

4. Holding period of the trade is one full trading day, so we open the trade in the beginning of the day, and we close it in the end of the trading day.

3.7. Back-Test

Back-testing is useful when we want to test our methods without having to wait for the markets. That why we decided to use it. The principle of back-testing involves testing the created strategy idea on the data from the past.

Historical performance does not secure future profits but can greatly contribute to strategy's sense of confidence. Back-testing was an application of created technique at a specific time span to see how the strategy could perform in the future.

3.8. Results of investing strategies

We empirically analyzed two created strategies in order to evaluate their profitability and performance. We searched for efficient pair trading strategies with usage of classic ETFs and Leveraged ETFs. In this subchapter, we provide a comparison of obtained outcomes.

Table 25: Results of first strategy using ETFs based on pair SPY/DJI

Statistic	
Tot Perf	21,89%
Max: BEST TRADE	2,86%
Min: WORST TRADE	-1,56%
#Trade	396
#Pos	221
#Neg	175

Source: Own elaboration

Table 26: Results of second strategy using LETFs based on pair SPY/DJI

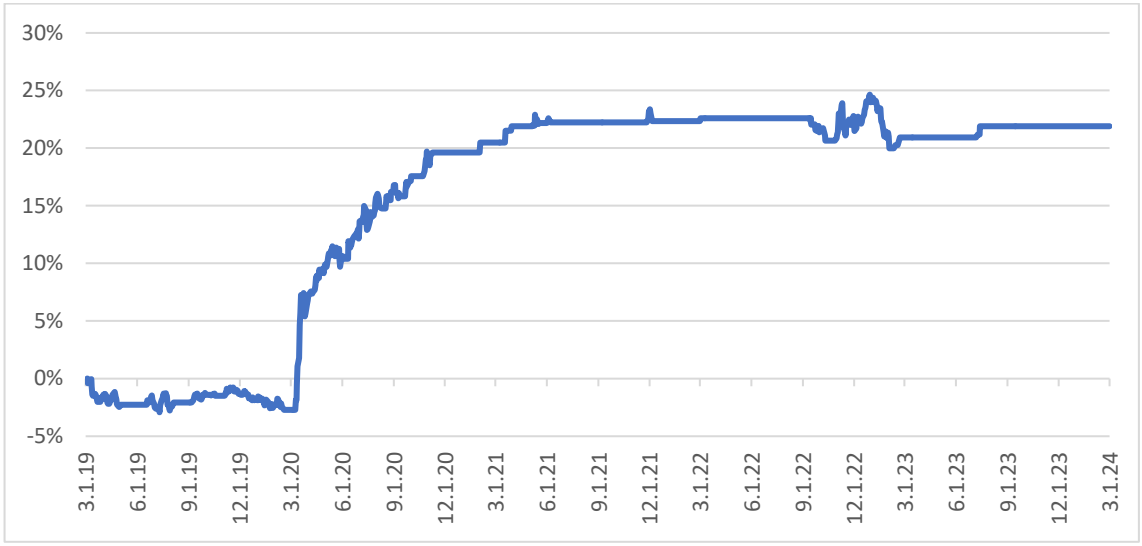
Statistic	
Tot Perf	30,81%
Max: BEST TRADE	20,04%
Min: WORST TRADE	-19,38%
#Trade	396
#Pos	196
#Neg	200

Source: Own elaboration

Obtained results that are shown in tables 25 and 26. Regarding to results, in the second strategy we executed same number of trades as in the first strategy. However, the number of negative trades was bigger in the second strategy (200) than in the first strategy (174).

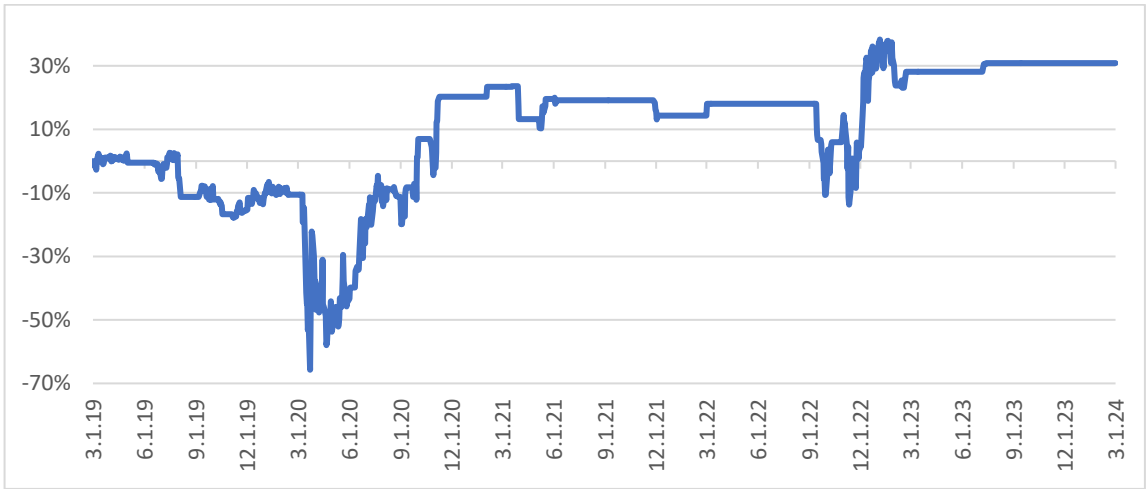
Both strategies based on the pair SPY/DJI revealed positive results. The total cumulative performance of the second strategy that was using Leveraged ETFs trading is 30.81%, which is 9.92%better than the total performance of the first strategy using simple ETFs. The second strategy's highest profit was 20.04% but it's also crucial to mention that the second method had a larger range between the highest gain and minimum loss, meaning that the worst trade resulted in a loss of -19.38%. In the other hand highest profit of the first strategy was 2.86% and the highest loss was 1.56%. This result confirms the fact that with the bigger leverage we can achieve higher profits but higher losses as well.

Graph 18: Cumulative performance of the first strategy using ETFs based on pair SPY/DJI



Source: Own elaboration

Graph 19: Cumulative performance of the second strategy using LETFs based on pair SPY/DJI



Source: Own elaboration

In summary, both trading strategies showed promising results in terms of performance, nevertheless, the second approach proves to be more efficient in terms of total returns. Even though the second strategy that was using LETFs showed better result, from the graph of cumulative performance we can see that in the period around March 2020, when the markets were volatile, the second strategy was performing badly, and cumulative performance was almost -70%. This finding confirms the fact that trading of LETFs in the period of high market volatility is very risky and requires good risk and money managements as a prevention against even higher loses.

As a next step, to obtain other results as next step we tested the strategies on the pair SPY/QQQ.

Table 27: Results of first strategy using ETFs based on pair SPY/QQQ

Statistic	
Tot Perf	29,97%
Max: BEST TRADE	2,74%
Min: WORST TRADE	-2,68%
#Trade	376
#Pos	219
#Neg	157

Source: Own elaboration

Table 28: Results of second strategy using LETFs based on pair SPY/QQQ

Statistic	
Tot Perf	-22,49%
Max: BEST TRADE	18,45%
Min: WORST TRADE	-19,19%
#Trade	376
#Pos	204
#Neg	172

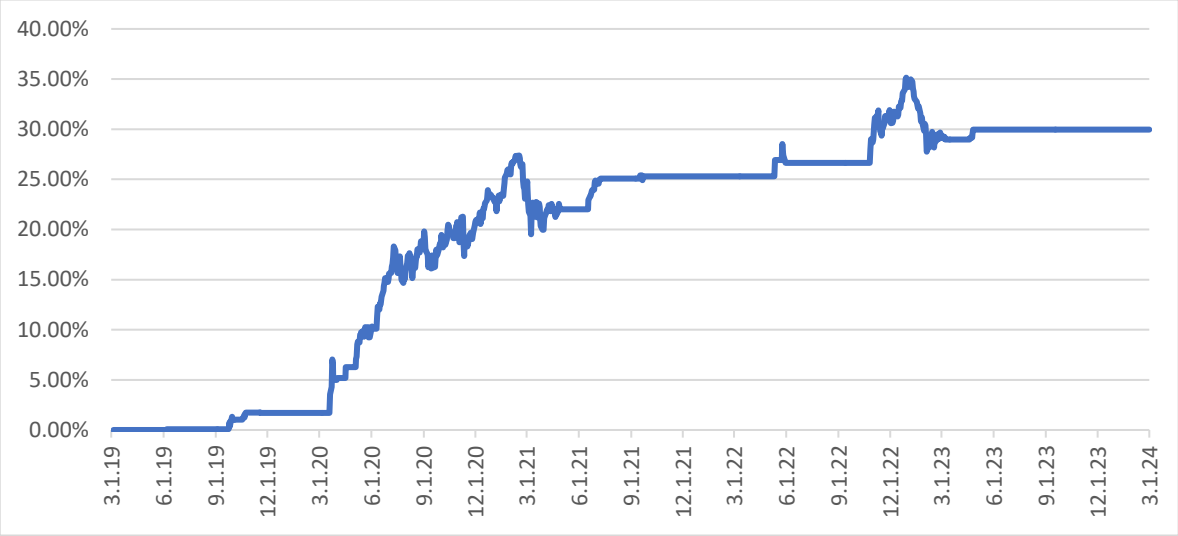
Source: Own elaboration

When we are comparing the outcomes shown in tables 27 and 28 of both strategies based on the pair SPY/QQQ, notable differences become apparent. A respectable overall performance of 29.97% was shown by the first strategy using ETFs which has a maximum daily profit of 2.74% and a maximum daily loss of -2.68%.

The first strategy maintained a larger proportion of positive trades (219) compared to negative trades (157), indicating a generally effective trading technique.

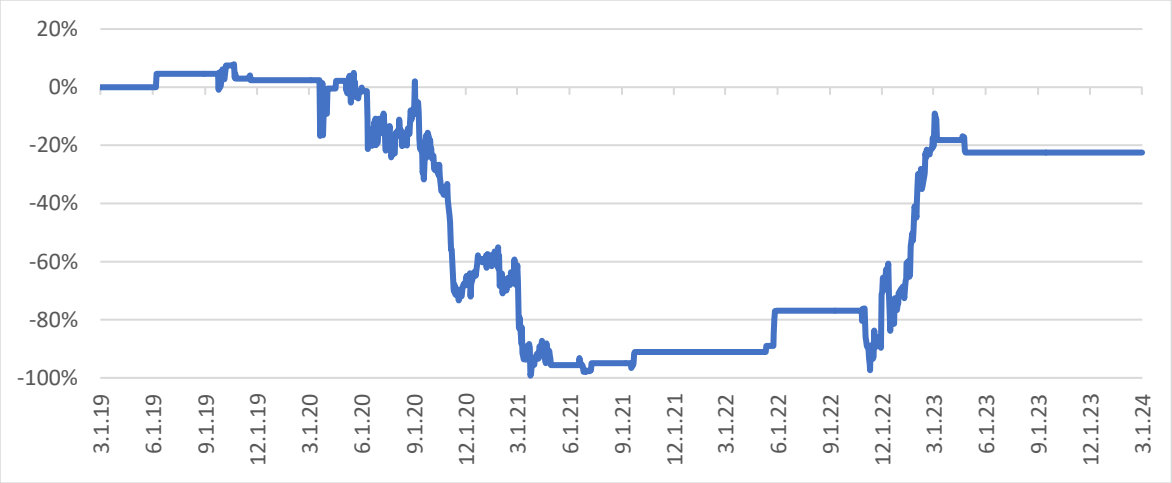
On the other hand, the second strategy using Leveraged ETFs showed loss of 22,49%, which indicates significant losses during the observed time. The pair had a large range of losses, with the worst transaction yielding a loss of 19.19%, although achieving a maximum return of 18,45%. Even though the number of profitable transactions (204) is higher than the number of non-profitable transactions (172), the strategy was not profitable.

Graph 20: Cumulative performance of the first strategy using ETFs based on pair SPY/QQQ



Source: Own elaboration

Graph 21: Cumulative performance of the first strategy using LETFs based on pair SPY/QQQ



Source: Own elaboration

Ultimately, the first strategy using ETFs showed positive overall performance and a greater percentage of profitable transactions than the second strategy using LETFs, as it is shown in graphs 21 and 22. Second strategy's overall poor performance and large losses highlight the dangers and the risk which is present while using the LETFs. The cumulative performance of the strategy was close to -100% in the time of volatile markets through pandemic of COVID-19. The pair (SPY/QQQ) is not efficient in this our strategy with usage of LETFs.

As a third pair, we investigated strategies behaviour on the DJI/QQQ.

Table 29: Results of first strategy using ETFs based on pair DJI/QQQ

Statistic	
Tot Perf	33,52%
Max: BEST TRADE	2,86%
Min: WORST TRADE	-4,39%
#Trade	554
#Pos	295
#Neg	259

Source: Own elaboration

Table 30: Results of second strategy using LETFs based on pair DJI/QQQ

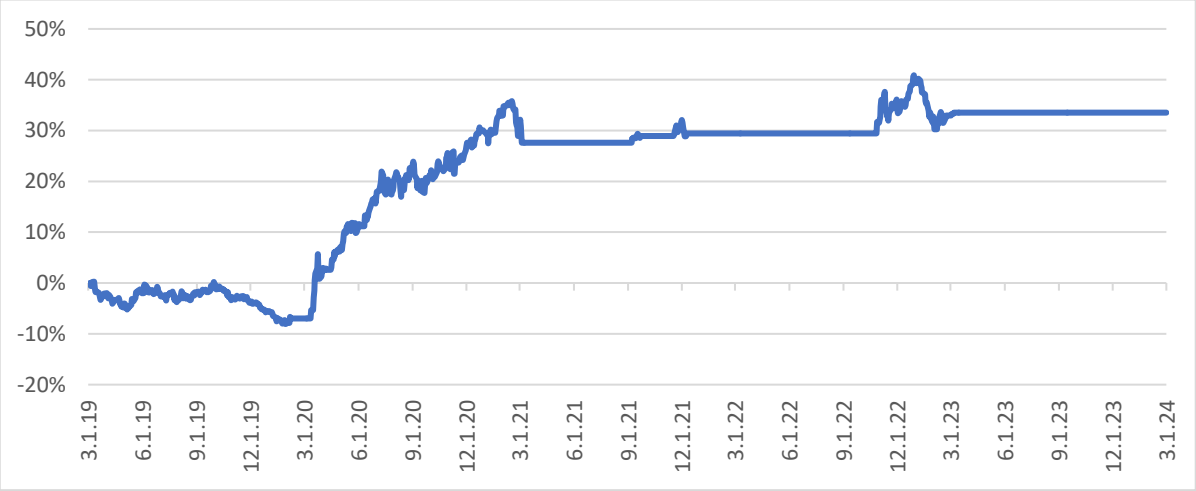
Statistic	
Tot Perf	2,01%
Max: BEST TRADE	21,72%
Min: WORST TRADE	-15,84%
#Trade	555
#Pos	258
#Neg	297

Source: Own elaboration

There are several distinctions between the strategies' outcomes showed in tables 29 and 30. The total performance of the first strategy using ETFs trading is 33.52%, which was much higher than the total performance of the second strategy using LETFs trading (2.01%). The first method had a larger proportion of positive transactions (295) compared to negative trades (259) while performing a comparable number of trades (554). This result suggests that the first strategy was regularly effective in trading than the second one. With a maximum loss of 4.39% and a maximum gain of 2.86%, the first approach exhibits a reasonably balanced risk-reward profile.

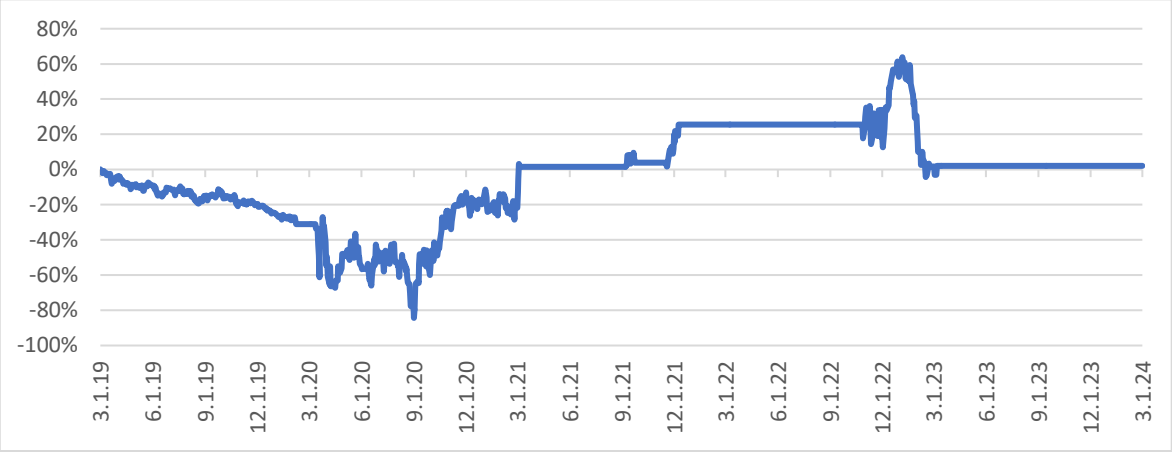
On the other hand, the second strategy showed a relatively poor overall performance of 2.01%. This strategy had a high maximum return of 21.72%, and the worst transaction resulted in a loss of 15.84%. Furthermore, based on the results and compared to the first approach, in this strategy there were more negative trades (297) than positive trades (258), indicating less consistency in performance and higher volatility.

Graph 22: Cumulative performance of the first strategy using ETFs based on pair DJI/QQQ



Source: Own elaboration

Graph 23: Cumulative performance of the first strategy using LETFs based on pair DJI/QQQ



Source: Own elaboration

In summary, the first strategy showed better overall performance and a larger percentage of profitable transactions. Significant number of trades that ended up negative and strategy's overall performance, indicated that the trading pair DJI/QQQ is not very efficient in our second strategy, but still profitable despite to obtained results.

Table 31: Comparison of LETFs strategy's performance

Statistic	SPY/DJI	SPY/QQQ	DJI/QQQ
Tot Perf	30,81%	-22,49%	2,01%
Max: BEST TRADE	20,04%	18,45%	21,72%
Min: WORST TRADE	-19,38%	-19,19%	-15,84%
#Trade	396	376	555
#Pos	196	204	258
#Neg	200	172	297

Source: Own elaboration

To summarize and compare obtained results we have constructed Table 31. showing the results of the strategies that are using the trading of LETFs. The best result was obtained on the pair SPY/DJI, where we were using trading of 3USL, 3USS, UDOW and SDOW. This result even overcome the results of trading classic ETFs in the same pair. Strategy that showed best performance had highest correlation between underlying indexes, in comparison to other two pairs, where the correlation was lower. High correlation is the main and key element for pair trade strategy to work, and our obtained results confirms this assumption.

On the other hand, we can say that the strategy based on the SPY/QQQ and DJI/QQQ showed lower effectivity, not just because of lower correlation between the pairs, but another potential reasons of low performance could be tracking errors of LETFs, non-equivalence, ineffective holding period or trading period. Our assumption was based on the historical data where we didn't consider other factors that could potentially influence effectivity of the strategy that is using LETFs. Another fact that we didn't consider is presence of costs such as bid-ask spreads, commissions, and financing costs. This would have influence on the profitability as well.

Even though pairs trading arbitrage is a sophisticated strategy that requires a good understanding of the markets, statistical analysis, and, often, the use of algorithmic trading systems to execute trades quickly, we can sum up that our strategy was efficient, and would bring results to the investors. Even though market conditions can change, and pairs that have historically moved together may diverge permanently due to the changes in market dynamics or individual circumstances of the assets involved. Therefore, careful risk management and continuous strategy evaluation are crucial for investors engaged in these types of strategies like ours.

Before using either of provided strategies in real-time trading conditions, we recommend doing further investigation, alongside with simulations of future market performance under different circumstances and investigate the behavior and results of the strategy under conditions of high volatility and market downfalls. After conducting such investigation there opens a possibility to improve and optimize the strategy. Another improvement can be done in risk management or analyzing different assets and find pairs with even higher correlation. Investors should carefully weigh the risks involved as the usage of LETFs is a lot riskier than usage of simple ETFs.

Apart from our usage in trading strategy, there are many other ways of utilizing LETFs discussed in next subchapter.

3.9. Other ways of using LETFs

As many investors are looking for new effective forms of using of Leveraged ETFs, they gained significant attention as an investment vehicle for hedging against different market scenarios. Although offering leveraged exposure to underlying assets or benchmarks is main function of LETFs, they can be applied in other ways to achieve certain financial goals.

To take advantage of temporary market opportunities or to dynamically modify portfolio allocation in reaction to shifting market circumstances, LETFs can be included into tactical asset allocation strategies. Through prudent fund allocation to leveraged exchange-traded funds (LETFs) with targeted ratios, investors may optimize returns during bull markets while reducing risk by making appropriate changes.

Leveraged exchange-traded funds (ETFs) may enhance the benefits of diversity in an investing portfolio by offering exposure to asset classes or sectors that are not easily accessible directly. By gaining leveraged exposure to various sectors, commodities, or geographical areas through LETFs, investors may diversify their holdings and perhaps increase risk-adjusted returns for short period of time.

For investors that want to reduce exposure to unfavorable market fluctuations or manage portfolio risk, LETFs may provide viable hedging options. Investors can protect their assets during volatile or uncertain times by devoting a portion of their portfolio to inverse LETFs as a strategic hedge against potential downside risk or market downturns.

LETFs can be used as a speculative investment vehicle for aggressive trading methods used by investors with higher risk tolerance and a speculative perspective who want to profit from short-term price swings or market inefficiencies. Although they come

with more risk and volatility, leveraged and inverse LETFs allow traders to increase profits or profit from expected market downfalls.

To sum, the utilization of LETFs by investors, is determined by a many of factors and depends on their investing strategy, including their investment goals, risk tolerance, market outlook, and familiarity with leveraged products.

Conclusion

Leveraged exchange-traded funds are a significant financial innovation of recent years and they provide various interesting opportunities for investors. The main aim of our thesis was to examine the efficiency of LETFs from investors perspective.

To achieve the goal, on the representative sample of ETFs and LETFs we have investigated efficient holding periods for LETFs an compared the results them with simple ETFs results. We have conducted statistical, correlation and regression analysis done in Excel.

The results of the analysis for chosen timespan confirmed that efficiency of LETFs in some cases can be higher, but most of the times for longer holding periods LETFs are not very efficient. This analysis would require deeper investigation withing more various assets to make clear conclusion.

Regarding to one of subgoals we created and tested strategy that is using LETFs as a trading instrument. The revealed results say that when certain conditions are fulfilled, the LETFs trading can provide even higher efficiency and performance for investors than the same trading strategy using simple ETFs. On the other hand, we must mention that developed strategy should be explored more, to secure even higher efficiency and higher profit for investors. Our recommendation is to perform simulations of future market movements and investigate the behavior and results of the strategy conditions of high market volatility and market downfalls. Secondly, we suggest that future work should be focused on usage of different type of data, adding more variables to the models and simulations or to use a completely different assets in the model, which could lead to better results.

To fulfill subgoals of the work we have focused on different features of LETFs. We have examined the advantages and disadvantages of leveraged exchange-traded funds, such as their reduced expenses, ease of use, openness, liquidity, and tax efficiency, the study provides valuable information for investors seeking to optimize their investment plans. In the practical part of the work, we have conducted and explained actual trends in the market with LETFs and discussed the potential of effective usage following by comparison of LETFs and classic ETFs, its performance, holding periods, and efficiency within same market conditions. At the end of the practical part, we have discussed obtained results and other possible ways of effective usage of Leveraged ETFs, which could be the subject of future investigation.

In conclusion, we can sum up, that Leveraged ETFs provide unique possibility for investors to increase their profits, if they are used properly. Despite of the results we obtained from the created strategy when specific conditions are met LETFs can be even more efficient than simple ETFs, but in more cases simple ETFs will deliver better performance. On the other hand, we must mention that trading of leveraged assets is connected not only to potential of higher profits, but also to potential of higher losses. Investment into LETFs always assumes higher risk and it is up to investors and their risk preferences whether they want to accept that risk for potential higher returns.

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