



UNIVERSITÀ  
DI PAVIA

**Dipartimento di Scienze Economiche e Aziendali**

**Corso di Laurea Magistrale in Finance**

---

**Empirical performances of a portfolio scenarios  
simulation mechanism based on a two-factor  
industry index model**

**Relatore:**

**Chiar.mo Prof. Riccardo Brignone**

**Tesi di Laurea  
di Martina Baltazzi**

**Matr. n. 562690**

**Anno Accademico 2025-2026**

## **Abstract (English)**

This thesis evaluates how effectively a return simulation mechanism can be employed to develop and optimize stock portfolios. The analysis starts from the Single-Index Model, which assumes that stock returns are driven by one market factor and then extends this approach using a two-factor Multi-Index Model, which also considers the systematic effects of each stock's industry sector; the scenarios generated by both models are used in a Mean-CVaR optimization process and assessed through a rolling out-of-sample backtest. The results indicate that sectoral information gives a clearer picture of the relationships between stock returns and leads to differences in portfolio composition and risk; overall, the Multi-Index strategy achieves better performance and risk-adjusted results than the Single-Index strategy and comparing both strategies with market benchmarks and other portfolio optimization methods further helps assess the robustness and competitiveness of the proposed approach.

## **Abstract (italiano)**

**Titolo: Performance empiriche di un meccanismo di simulazione di scenari di portafoglio basato su un modello a due fattori con indice settoriale**

Questa tesi esamina l'efficacia di un meccanismo di simulazione di scenari di rendimento per la costruzione e l'ottimizzazione di portafogli azionari; l'analisi parte dal Single-Index Model, che presuppone che i rendimenti azionari dipendano da un unico fattore, rappresentato dal mercato, ed estende questo approccio attraverso il Multi-Index Model a due fattori, che considera anche gli effetti sistematici del settore industriale di ciascun titolo; gli scenari generati da entrambi i modelli vengono utilizzati in un processo di ottimizzazione Mean-CVaR e valutati tramite un backtest a finestra mobile su dati non utilizzati nella fase di stima. I risultati indicano che le informazioni settoriali forniscono un quadro più chiaro delle relazioni tra i rendimenti azionari e portano a differenze nella composizione del portafoglio e nel rischio; nel complesso, la strategia basata sul Multi-Index Model ottiene una migliore performance in termini di rendimento corretto per il rischio rispetto all'approccio basato sul Single-Index Model, inoltre il confronto di entrambe le strategie con i benchmark di mercato e altri metodi di ottimizzazione del portafoglio contribuisce ulteriormente a valutare la robustezza e la competitività dell'approccio proposto.

## Table of contents

|                                                                                |    |
|--------------------------------------------------------------------------------|----|
| <b>Introduction</b>                                                            | 4  |
| <b>1 Theoretical Foundation of the Single Index Model</b>                      | 5  |
| 1.1 Theoretical introduction to the Single Index Model                         | 5  |
| 1.2 Dataset and Return Preparation                                             | 6  |
| 1.3 Theoretical assumptions and estimation of the model parameters             | 10 |
| 1.3.1 Alpha: autonomous return component                                       | 12 |
| 1.3.2 Beta: sensitivity to market movements                                    | 12 |
| 1.3.3 Residual component                                                       | 14 |
| 1.3.4 $R^2$ as a Measure of Explanatory Power                                  | 15 |
| 1.4 Economic Interpretation of the Results                                     | 16 |
| 1.4.1 Distribution of Betas                                                    | 17 |
| 1.4.2 Variance decomposition                                                   | 18 |
| 1.4.3 Distribution of $R^2$                                                    | 20 |
| 1.5 Residual Correlation Analysis                                              | 22 |
| <b>2 Portfolio Optimization with Mean-CVaR Based on the Single-Index Model</b> | 25 |
| 2.1 Example month                                                              | 26 |
| 2.1.1 Estimation Window and SIM estimation                                     | 27 |
| 2.1.2 Scenario generation                                                      | 28 |
| 2.1.3 Mean-CVaR optimization problem                                           | 30 |
| 2.1.4 Portfolio weights                                                        | 33 |
| 2.1.5 Out-of-Sample Realized Return                                            | 37 |
| 2.1.6 Interpretation and conclusions                                           | 40 |
| 2.2 Rolling Backtest of the SIM Mean-CVaR Portfolio Strategy                   | 42 |
| 2.2.1 Portfolio Performance and Downside Risk Dynamics                         | 43 |
| 2.2.2 Scenario Distributions and Realized Returns                              | 47 |
| 2.2.3 Sector Allocation Dynamics                                               | 51 |
| 2.2.4 Weight Concentration and Portfolio Breadth                               | 53 |
| 2.2.5 Discussion of results                                                    | 55 |
| <b>3 Theoretical Foundation of the Multi-Index Model</b>                       | 58 |
| 3.1 Introduction and Motivation for the Multi-Index Model                      | 58 |
| 3.2 Methodological Setting of the Multi-Index Model                            | 59 |
| 3.3 Comparison of sector correlation structure: SIM vs MIM                     | 62 |
| 3.4 Sectoral Dependence and Portfolio Diversification                          | 65 |
| <b>4 Portfolio Optimization with Mean-CVaR based on the Multi-Index Model</b>  | 67 |
| 4.1 Scenario Generation under the Multi-Index Framework                        | 67 |
| 4.1.1 Estimation Results under the Multi-Index Model                           | 68 |
| 4.1.2 Stock-Level Scenario Comparison: SIM vs MIM                              | 69 |
| 4.1.3 Simulated Portfolio Return Distribution under the MIM                    | 72 |

|          |                                                                    |            |
|----------|--------------------------------------------------------------------|------------|
| 4.2      | Mean–CVaR Portfolio Construction under the Multi-Index Model ..... | 73         |
| 4.2.1    | Optimal Portfolio Weights .....                                    | 74         |
| 4.2.2    | Sector Allocation of the MIM Portfolio .....                       | 75         |
| 4.2.3    | Risk Characteristics of the Optimized Portfolio .....              | 77         |
| 4.3      | Realized Performance Evaluation .....                              | 78         |
| 4.3.1    | Realized Monthly Return Distribution of the MIM Strategy .....     | 78         |
| 4.3.2    | Cumulative Performance and Drawdown Comparison: SIM vs MIM .....   | 80         |
| 4.3.3    | Summary Performance Comparison .....                               | 84         |
| <b>5</b> | <b>Benchmark comparison</b> .....                                  | <b>86</b>  |
| 5.1      | Introduction and Benchmark Strategies .....                        | 86         |
| 5.1.1    | S&P 500 .....                                                      | 87         |
| 5.1.2    | Equal Weight .....                                                 | 88         |
| 5.1.3    | Mean–Variance .....                                                | 89         |
| 5.1.4    | Minimum Variance .....                                             | 90         |
| 5.1.5    | Minimum CVaR .....                                                 | 91         |
| 5.2      | Comparative analysis of portfolio strategies .....                 | 92         |
| 5.2.1    | Mean-Risk Comparison .....                                         | 92         |
| 5.2.2    | Minimum-Risk Comparison .....                                      | 95         |
| 5.2.3    | Discussion of Results .....                                        | 99         |
|          | <b>Conclusion</b> .....                                            | <b>102</b> |
|          | <b>Bibliography</b> .....                                          | <b>104</b> |

## Introduction

This thesis evaluates the effectiveness of a return-scenario simulation mechanism for portfolio construction and optimization, focusing in particular on the role of the dependence structure between asset returns; the analysis begins with the Single-Index Model (SIM), which offers a parsimonious description of return co-movements through exposure to a common market factor and then extends this to a two-factor Multi-Index Model (MIM), incorporating sector information alongside the broader market component. The primary objective is to determine whether a more detailed representation of systematic dependencies produces more accurate simulated return scenarios and supports more effective portfolio decisions. To address the research question empirically, the thesis combines return scenario generation with Mean-CVaR portfolio optimization and the performance of the resulting portfolios is assessed through a rolling out-of-sample backtest; the empirical analysis covers 359 U.S. equities over the 2007–2021 period, moreover the S&P 500 is used as the market factor, while GICS classifications are employed to identify the sectoral exposures of the stocks.

The thesis is structured as follows: chapter 1 introduces the Single-Index Model (SIM) and analyses its empirical properties, focusing on the dependence between asset returns that is not explained by the market factor; chapter 2 uses the SIM to generate return scenarios and incorporates them into a Mean-CVaR portfolio optimization process and the resulting strategy is then evaluated out of sample; chapter 3 presents the Multi-Index Model (MIM), which incorporates sector information, and examines whether this extension provides a more accurate representation of return dependencies; chapter 4 applies the same scenario-generation and optimization procedure to the MIM and compares the resulting portfolios with those obtained using the SIM. Finally, chapter 5 compares the proposed strategies with the S&P 500, an equally weighted portfolio, and alternative optimization methods under both mean-risk and minimum-risk settings.

# **1 Theoretical Foundation of the Single Index Model**

## **1.1 Theoretical introduction to the Single Index Model**

Modern portfolio theory, developed by Markowitz, constitutes a key turning point in quantitative finance, as it casts the portfolio selection problem in terms of expected return and risk (Markowitz, 1952). Nonetheless, its empirical application faces a significant practical constraint: the requirement to estimate a very large number of covariances across the securities in the investment universe; specifically, in a portfolio of  $N$  assets, the count of distinct pairwise covariances increases as  $N(N - 1)/2$ , which makes the estimation process progressively more demanding and prone to instability as the number of assets grows (Elton et al., 2014).

It is precisely to address this issue that the Single Index Model (SIM) was proposed by Sharpe (1963): the model's main premise is that the joint movement of individual stock returns can be attributed to a single shared systematic driver, namely the overall market return. Rather than estimating each pairwise covariance directly, the SIM posits that every asset is connected to the market through a sensitivity parameter, typically referred to as beta; the return on security  $i$  is then described as the sum of three elements: a constant term, the component attributable to market fluctuations, and an idiosyncratic error term that captures firm-specific effects; the model's economic rationale indicates that price changes primarily arise from common exposure to macroeconomic conditions, investor sentiment, expectations about the business cycle, monetary circumstances, and broad market risk factors captured by the market index. In this view, the market acts as the main source of return co-movement, whereas idiosyncratic shocks are confined to the residual component, a perspective yielding a clear partition of total risk into systematic risk, measured by beta, and unsystematic risk, captured by the residual variance (Elton et al., 2014).

The Single-Index Model is primarily valued for its parsimony, because rather than estimating the full covariance matrix, the analyst can proceed by estimating, for

each asset, the parameters alpha and beta, the residual variance, and the variance of the market index; this approach substantially lowers the amount of data required and reduces estimation error, particularly in situations where the number of assets is large. Accordingly, Elton, Gruber, Brown, and Goetzmann, (2014) highlighted the practical importance of index models as efficient instruments for approximating correlation patterns in portfolio management.

Another key consequence relates to diversification: because idiosyncratic residual risk can be mitigated by holding a large number of securities, the remaining non-diversifiable risk primarily reflects exposure to overall market movements, as a consequence, beta becomes a fundamental measure of the extent to which a stock influences portfolio risk, indeed stocks with beta greater than one generally intensify market fluctuations, while those with beta below one exhibit lower sensitivity to aggregate changes (Elton et al., 2014).

Despite the subsequent development of more advanced multifactor frameworks, the Single Index Model remains highly pertinent: it continues to serve as a common reference point due to its clarity, ease of interpretation, and modest computational requirements, in addition, across empirical portfolio construction, risk decomposition, and educational contexts, it is still regarded as one of the most transparent connections between statistical modelling and economic reasoning. Consequently, the SIM may be understood not only as a simplification of Markowitz, but also as one of the most consequential parsimonious approaches in contemporary asset management (Sharpe, 1963).

## 1.2 Dataset and Return Preparation

This study is grounded in an empirical panel comprising 359 equities listed in the United States, for which daily trading prices are observed over the period from December 2007 to March 2021. For each company, the dataset reports its GICS classification, enabling a consistent assignment of firms to a sector and industry segment throughout the entire sample horizon. In addition, the S&P 500 Index is included as an explanatory benchmark and is treated as a proxy for the market

portfolio within the subsequent application of the Single Index Model (Sharpe, 1963). Taken together, these features generate an internally coherent empirical framework in which the contemporaneous co-movements of individual securities can be examined jointly with concurrent market-wide fluctuations.

Very useful for our analysis is the Global Industry Classification Standard (GICS), originally developed by MSCI and S&P. The GICS's objective is to provide a schematic and organized framework for categorizing companies based on their principal business activities. Specifically, this framework simplifies and improves comparability across companies, enabling greater efficiency and transparency in portfolio construction and equity analysis using a common taxonomy. Indeed, the GICS has a distinctly hierarchical structure: initially, sector groupings tend to be broader, subsequently narrowing into multiple, more specific categories, namely sectors and subsectors. This organization allows for a wide variety of assessments, from more detailed ones at the lower levels of the hierarchy to more general and aggregate ones at the higher levels of the hierarchy. This allows for the capture of both overall industry effects and subtleties at the individual company level (MSCI & S&P Dow Jones Indices, 2024).

From an empirical standpoint, this classification is meaningful because firms within the same industry are typically subject to comparable demand conditions, cost structures, regulatory requirements, and technological disturbances (Chan et al., 2007). Bhojraj, Lee and Oler (2003) find that GICS groupings are particularly effective in capturing stock return co-movements and in isolating economically similar peer firms, often performing better than conventional SIC-based schemes (Bhojraj et al., 2003). This evidence therefore justifies the use of GICS labels in the present dataset, in addition, segmenting firms according to GICS is expected to enhance the stability of the estimated factor relationships.

As for the sample period, it is particularly informative because it covers multiple, clearly differentiated market conditions, specifically it begins during the latter stages of the 2007–2009 Global Financial Crisis, proceeds through the prolonged post-crisis bull phase, incorporates episodes of elevated volatility, including the

2018 correction, and ends with the sudden disruption brought about by the COVID-19 market shock in 2020. Consequently, the dataset includes both benign and adverse environments, which enables robust evaluation of risk models and portfolio allocation approaches (Jiang et al., 2024).

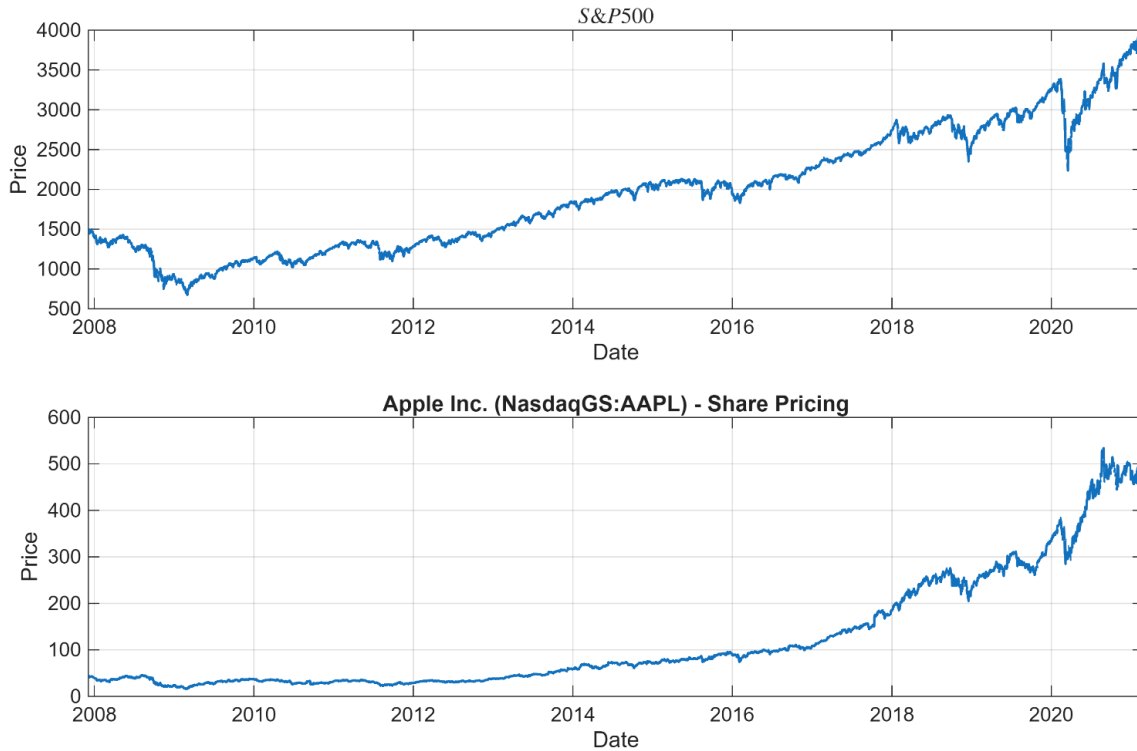
Returns are derived from daily price observations by taking percentage changes between consecutive dates, which converts raw price levels into a more stationary measure of performance (Campbell et al., 1998); they are computed using aligned trading days, and any missing observations are handled in a consistent manner across the panel, so that the estimation horizon is defined uniformly for all assets; let  $P_{i,t}$  represent the price of asset  $i$  at time  $t$ , and define the simple return as:

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}$$

This procedure is commonly adopted in empirical finance because asset prices may exhibit non-stationarity and are not directly comparable across firms that operate with different nominal scales; by contrast, returns are expressed in a dimensionless form, which allows the gain or loss over the interval to be interpreted straightforwardly (Campbell et al., 1998).

The same procedure is carried out for the S&P 500 Index, producing the market return series later used as the common explanatory variable in the SIM (Sharpe, 1964); both individual securities and the benchmark index are represented using a consistent format, which facilitates regression-based estimation of alpha, beta, and idiosyncratic risk components (Sharpe, 1963).

*Figure 1.2.1 - Historical price dynamics of the S&P 500 and Apple Inc*



*Source: author's elaboration using MATLAB outputs.*

Figure 1.2.1 confirms the heterogeneity of the sample, for illustrative purposes, the first chart, which represents the evolution of the S&P 500 index's price levels, highlights the sharp declines observed during the 2008 financial crisis and the sharp correction associated with the 2020 pandemic shock. In both cases, the subsequent recovery is characterized by a rapid rebound, followed by a period of stabilization, illustrating the ability of market processes to transmit risk and thus restore growth dynamics. The second chart, focusing on Apple Inc., further highlights that individual stocks may not accurately reflect index-level movements: although Apple also exhibits periods of pronounced fluctuation, its overall long-term trajectory remains relatively stronger, with growth that is not fully explained by the overall market trend and is accompanied by volatility that differs in both magnitude and timing. This divergence is consistent with the core principle of index models, according to which each asset's return can be decomposed into a systematic component, determined by market factors, and an idiosyncratic component specific to each individual company (Sharpe, 1963;

Sharpe, 1964). Consequently, even when the systematic component of returns moves in parallel with the index, residual idiosyncratic effects allow each security to exhibit a unique performance and risk profile (Elton et al., 2014).

The dataset is then structured around three key aspects: cross-sectional coverage of 359 stocks, extensive time-series information comprising over thirteen years of daily observations that reflect substantial variability under different market conditions, and sectoral characterization through GICS labels, which permits the direct assignment of each asset to a comparable industry category; these attributes offer a solid empirical basis for estimating covariance relationships with greater robustness, while informing the specification and calibration of index models that account for sectoral structure and supporting the portfolio optimization methods outlined in the following chapters (Bhojraj et al., 2003; Elton et al., 2014).

### 1.3 Theoretical assumptions and estimation of the model parameters

According to the single-index model, a stock's return is composed of two components: a systematic component that varies in tandem with general market movements and an idiosyncratic component that reflects firm-specific influences not considered by the market (Elton et al., 2014, Sharpe, 1964); the model parameters, in the empirical analysis, are determined by estimating, for each stock, a linear association between its excess return and the excess return of the selected market proxy; as previously mentioned, this study adopts the S&P 500 index as the market benchmark (Sharpe, 1964), and the parameters are obtained by regressing the time series of individual stock returns on market returns.

The conventional specification is:

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + \varepsilon_{i,t}$$

where  $R_{i,t}$  denotes the return of asset  $i$  at time  $t$ ,  $R_{m,t}$  is the market return,  $\alpha_i$  is the intercept representing the portion of expected return not accounted for by market movements,  $\beta_i$  captures the asset's systematic risk (that is, its degree of co-movement with the market), and  $\varepsilon_{i,t}$  is the residual component that reflects

idiosyncratic shocks and other influences specific to asset  $i$ , assumed to be orthogonal to the market; this regression specification constitutes the practical foundation of the SIM, as it transforms observed return data into economically meaningful risk parameters, distinguishing systematic risk attributable to market returns from variation that is specific to each asset (Elton et al., 2014).

Let us delineate the SIM's underlying theoretical assumptions: by construction, the first condition requires that the residual term has zero mean:

$$E(e_i) = 0$$

additionally, it is assumed that the residual term is independent of the market factor:

$$E[e_i(R_m - \bar{R}_m)] = 0$$

where:

- $R_m$  represents the market index of the return;
- $\bar{R}_m$  identifies the market average return over the sample period.

This condition guarantees that the residual term reflects only the remaining variation not accounted for by the market index.

The Single-Index Model relies on the simplifying assumption that the residual terms are mutually independent across securities:

$$E(e_i e_j) = 0 \quad i \neq j$$

under these assumptions, the variance of a single security can be separated into two different components:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{e_i}^2$$

where:

- $\beta_i^2 \sigma_m^2$  identifies systematic risk, namely the portion of volatility attributable to exposure to the market factor.
- $\sigma_{e_i}^2$  denotes idiosyncratic risk linked to firm-specific shocks.

Similarly, the covariance between two securities is fully attributable to their shared exposure to the market factor:

$$\sigma_{ij} = \beta_i \beta_j \sigma_m^2$$

This outcome represents the core simplification provided by the SIM: rather than directly estimating a multitude of pairwise covariances, the interdependence among returns is modelled through a single shared systematic factor (Elton et al., 2014).

### 1.3.1 Alpha: autonomous return component

The parameter alpha denotes the fraction of a security's expected return that cannot be attributed to broad movements in the market: in regression-based terms, alpha is identified with the intercept of the security characteristic line (the independent component), meaning the expected excess return of the security when the market factor is equal to zero (Elton et al., 2014). A positive alpha implies that the stock, on average, delivers returns above the level predicted by its market exposure (beta), while a negative alpha indicates that it underperforms relative to the systematic-risk benchmark (Horenstein, 2021), consequently, alpha may be viewed as a firm-specific contribution to performance, capturing return drivers that are not reflected in the common market factor (Elton et al., 2014). Empirically, such a component may stem from managerial capability and strategic choices, differences in competitive position, variations in operational efficiency, innovation-related strengths, and other persistent characteristics that shape cash flows and influence investor expectations (Zurek & Heinrich, 2021). Although alpha is frequently interpreted as a sign of abnormal performance, it may also be affected by estimation uncertainty and by the adequacy of the chosen model; therefore, its meaning should be assessed alongside the statistical significance and the assumptions underpinning the regression framework (Hollstein et al., 2020).

### 1.3.2 Beta: sensitivity to market movements

The coefficient beta is the key parameter of the Single Index Model: it reflects the

sensitivity of a given stock to fluctuations in the market portfolio. Specifically, beta corresponds to the slope of the regression line and represents the incremental impact of market returns on the return of the security (Elton et al., 2014).

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)}$$

where:

- $R_i$  denotes the return on security  $i$ ;
- $R_m$  denotes the return of the market index;
- $Cov(R_i, R_m)$  indicates the extent to which the security's returns co-move with the market return;
- $Var(R_m)$  indicates the degree of dispersion, or risk, associated with market returns (Elton et al., 2014).

A beta of 1 indicates that a stock's returns generally move in the same proportion as overall market movements, while a beta above 1 is associated with a more aggressive, growth-oriented security, whose returns respond more strongly to changes in the market and thus tend to display higher volatility than the market, by contrast, a beta below 1 corresponds to a defensive security, implying that its returns are typically less affected by broad market shocks and exhibit relatively steadier variability. Although negative beta values are rare, they imply that the security's returns tend to move in the opposite direction to the market, which may confer some hedging benefits during unfavourable market conditions (Elton et al., 2014).

Blume (1971) argued that the estimated betas observed in empirical data are not strictly constant through time; rather, they display systematic temporal behaviour, notably a tendency to regress toward the market's long-run average in later periods; this mean-reversion feature is particularly consequential for empirical finance, because it suggests that beta estimates computed from historical return outcomes may capture short-lived conditions rather than the enduring, systematic

component of risk that underpins investors' required returns. As a result, relying mechanically on past beta estimates can lead to forecast inaccuracies, especially when market-wide risk premia and firms' underlying sensitivities shift over time; in portfolio management practice, this issue supports the adoption of adjusted or smoothed beta measures, which explicitly account for estimation uncertainty and the expectation that individual beta estimates move toward a long-term benchmark (Echterling et al., 2015; Blume, 1971).

### 1.3.3 Residual component

The residual component reflects the portion of returns that cannot be attributed to overall market fluctuations (Elton et al., 2014). In this context, it accounts for firm-specific shocks; examples include earnings announcements, litigation or regulatory developments, product launches and product mix changes, significant top management changes, unexpected shifts in corporate strategy, and idiosyncratic industry news affecting only the individual firm (Herskovic et al., 2016, Aabo et al., 2017). By definition, the residual component is assumed to have a mean of zero (so that, on average, it does not induce systematic abnormal performance) and to be uncorrelated with the market factor, thus separating firm-level deviations from the effects of aggregate market conditions (Elton et al., 2014).

This term's variance captures idiosyncratic risk:

$$\sigma_{\varepsilon_i}^2 = Var(\varepsilon_i)$$

where:

- $\varepsilon_i$  is the idiosyncratic error term for security  $i$ ;
- $Var(\varepsilon_i)$  represents the variance of this residual component;
- $\sigma_{\varepsilon_i}^2$  therefore, represents the idiosyncratic risk, that is, the part of the total risk specific to the individual firm and unrelated to market-wide fluctuations (Elton et al., 2014).

A high residual variance suggests that a large share of the stock's total uncertainty arises from idiosyncratic, firm-specific developments (such as managerial choices,

product outcomes, litigation results, and other company-level shocks) rather than from the broad market movements represented by common risk factors (Herskovic et al. 2016). Accordingly, this type of risk is viewed as diversifiable: investors holding a sufficiently large and well-diversified portfolio of stocks can mitigate the effect of these uncorrelated disturbances, thereby lowering their contribution to overall portfolio volatility. Instead, systematic market risk, driven by macroeconomic conditions and changes in general investor sentiment, remains unaffected by diversification (Elton et al., 2014, Markowitz, 1959).

#### 1.3.4 $R^2$ as a Measure of Explanatory Power

The coefficient of determination,  $R^2$ , indicates the fraction of the total variation in a stock's returns that can be explained by changes in the market factor; it reflects how well the Single Index Model accounts for the return behaviour of a particular security, providing a measure of the model's explanatory effectiveness for that asset (Elton et al., 2014).

$$R^2 = 1 - \frac{\sum \varepsilon_{i,t}^2}{\sum (R_{i,t} - \bar{R}_i)^2}$$

where:

- $R_{i,t}$  denotes the return of security  $i$  at time  $t$ ;
- $\bar{R}_i$  represents the average return for security  $i$ ;
- $\varepsilon_{i,t}$  represents the residual portion of the model, that is, the segment of returns not accounted for by the market factor;
- $\sum \varepsilon_{i,t}^2$  quantifies the portion of the security's return variability that remains unexplained;
- $\sum (R_{i,t} - \bar{R}_i)^2$  indicates the overall dispersion of the security's returns around their average value (Elton et al., 2014).

An  $R^2$  close to 1 suggests that movements in the market index explain nearly all of the variation in a stock's returns during the sample period, indicating a strong alignment between firm-level performance and broader aggregate conditions

(Sharpe, 1964); specifically, this result is commonly found among large, established companies whose revenues and trading activity closely track market dynamics and respond in a similar way to macroeconomic shocks. By contrast, a low  $R^2$  indicates that market-wide information has limited explanatory power (Elton et al., 2014) and stock returns are primarily shaped by firm factors, including earnings surprises, changes in strategic expectations, managerial actions, and sector-related developments (Aabo et al., 2017).

#### 1.4 Economic Interpretation of the Results

Following the discussion of the SIM's main parameters and underlying theoretical framework, the model is applied empirically to estimate the SIM coefficients for each stock in the dataset; in support of that the table below reports, for illustrative purposes, ten selected stocks together with their estimated parameters, namely alpha, beta, residual volatility, and the coefficient of determination ( $R^2$ ).

*Figure 1.4.1 - Illustrative Estimation Results of the Single Index Model*

| Stock                                         | GICS Sector            | Alpha   | Beta   | ResidualStd | R2     |
|-----------------------------------------------|------------------------|---------|--------|-------------|--------|
| Texas Instruments Incorporated (NasdaqGS:TXN) | INFORMATION TECHNOLOGY | 0,0002  | 0,6969 | 0,0119      | 0,3792 |
| The Hershey Company (NYSE:HSY)                | CONSUMER STAPLES       | -0,0003 | 0,8544 | 0,0167      | 0,3161 |
| Pool Corporation (NasdaqGS:POOL)              | CONSUMER DISCRETIONARY | 0,0002  | 1,1787 | 0,0199      | 0,3821 |
| Ventas, Inc. (NYSE:VTR)                       | REAL ESTATE            | 0,0006  | 0,8364 | 0,0174      | 0,2901 |
| Trimble Inc. (NasdaqGS:TRMB)                  | INFORMATION TECHNOLOGY | -0,0003 | 1,3686 | 0,0163      | 0,5561 |
| Invesco Ltd. (NYSE:IVZ)                       | CONSUMER STAPLES       | 0,0003  | 0,7136 | 0,0123      | 0,3730 |
| BXP, Inc. (NYSE:BOX)                          | REAL ESTATE            | 0,0003  | 0,6498 | 0,0136      | 0,2873 |
| QUALCOMM Incorporated (NasdaqGS:QCOM)         | INFORMATION TECHNOLOGY | 0,0003  | 1,1706 | 0,0140      | 0,5530 |
| The Procter & Gamble Company (NYSE:PG)        | CONSUMER STAPLES       | 0,0001  | 0,5876 | 0,0085      | 0,4572 |
| Hormel Foods Corporation (NYSE:HRL)           | CONSUMER STAPLES       | 0,0000  | 1,3239 | 0,0194      | 0,4518 |

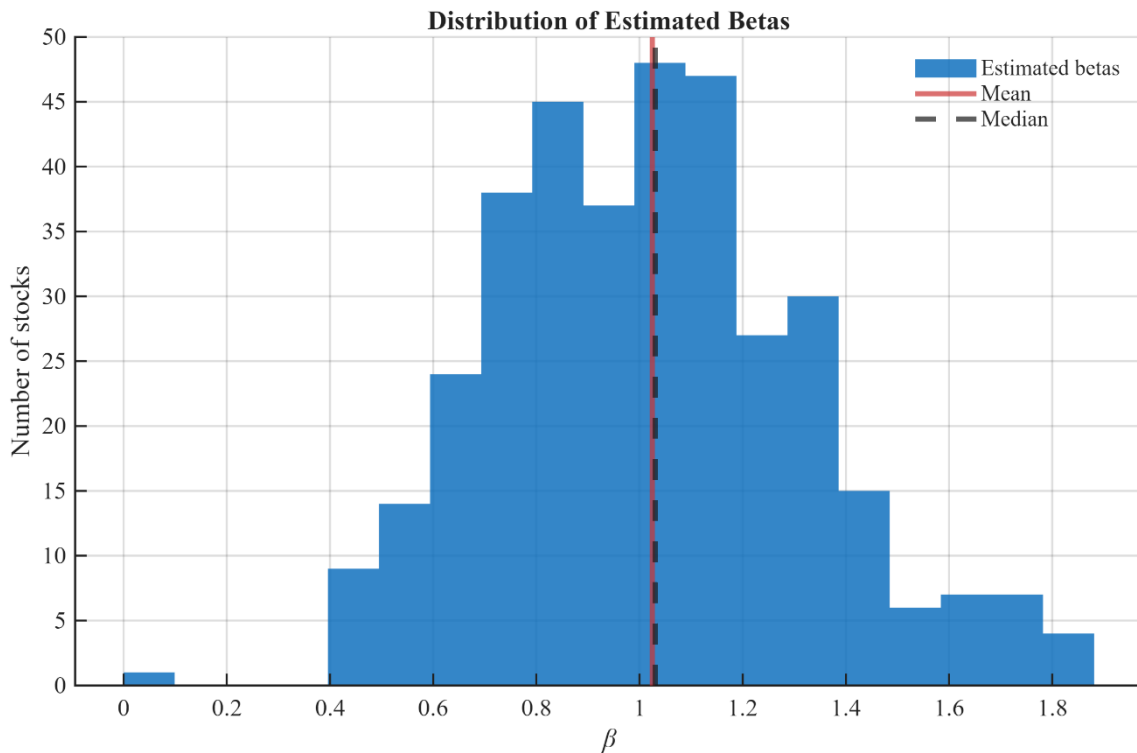
*Source: author's elaboration using MATLAB outputs*

After estimating the parameters of the single-index model, the subsequent step involves assessing their economic relevance using our sample of 359 U.S. stocks. In particular, the reported figures shed light on how market risk is apportioned across firms, the extent to which beta aligns with realized returns, and the proportion of overall volatility attributable to common sources relative to idiosyncratic firm-specific components.

### 1.4.1 Distribution of Betas

Building on the previous section's theoretical discussion of the beta coefficient and its function within the Single Index Model, the analysis below shifts from abstraction to empirical evaluation. Specifically, it considers the empirical distribution of the estimated betas derived from the stock sample used in this study: this approach yields a clearer understanding of the extent of systematic risk embedded in the dataset and highlights the degree of variation in firms' market sensitivities.

*Figure 1.4.2 – Distribution of estimated Betas*



*Source: author's elaboration using MATLAB outputs.*

As illustrated in the figure, the estimated beta distribution clusters around values close to one, with both the mean and the median positioned near the centre of the distribution. This pattern indicates that, on average, the stocks in the sample exhibit market sensitivity broadly aligned with the benchmark index. Put differently, the typical security in the dataset tends to adjust in proportion to overall market

movements (Elton et al., 2014; Sharpe, 1964), which is consistent with the large-cap U.S. equity segment underlying the analysis.

In addition the histogram reveals a noticeable dispersion around this central tendency, indeed several securities have beta estimates substantially above one, which implies an above-average exposure to systematic risk and, accordingly, a stronger responsiveness to market fluctuations; from an economic point of view, such firms are often associated with more cyclical or growth-oriented business models, whose returns typically magnify market dynamics across both expansionary and downturn periods.

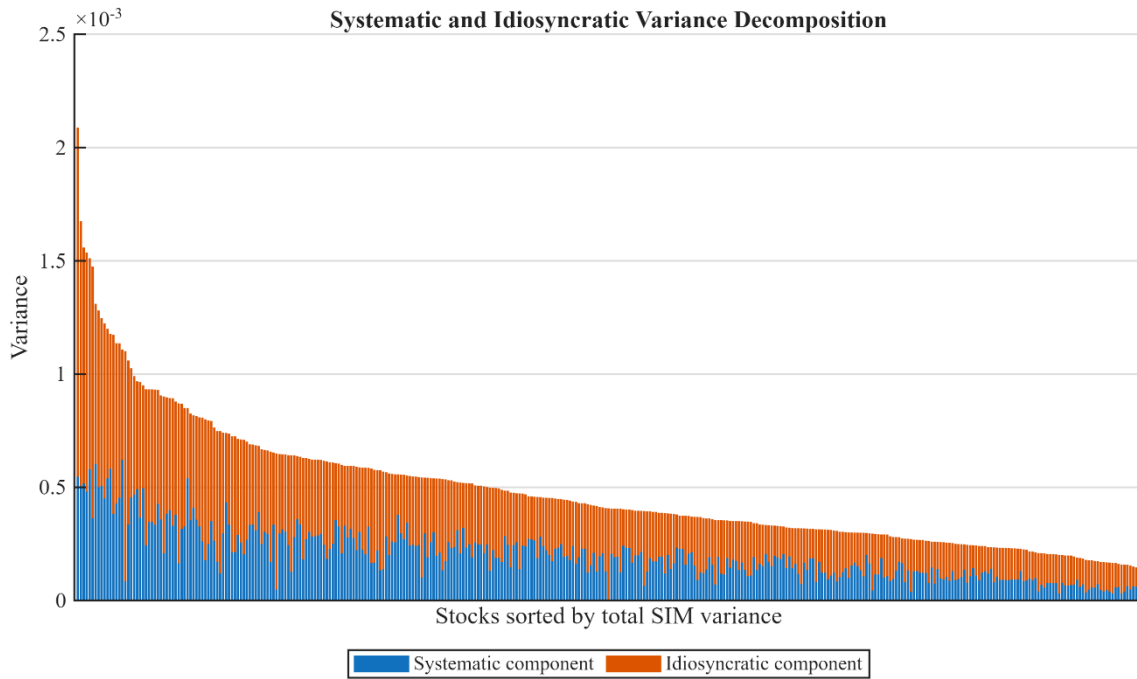
In contrast, a smaller group of securities shows betas below one, indicating comparatively defensive characteristics and reduced sensitivity to aggregate shocks (Kim, 2015; Sharpe, 1964). Another feature evident in the distribution is the limited occurrence of extreme beta estimates (Hollstein, 2020): most observations fall within a relatively moderate interval, whereas only a few securities present unusually high or very low beta values; consequently, this result suggests that the sample is not overly concentrated in highly speculative assets but instead reflects a fairly diversified equity universe with heterogeneous levels of systematic exposure (Blume, 1971).

Figure 2.4.2 illustrates how heterogeneity in beta coefficients is important for portfolio construction, as it allows investors to identify securities with different sensitivities to market movements and to set portfolio risk exposure accordingly.

#### 1.4.2 Variance decomposition

In the preceding sections, the residual component and its corresponding variance were introduced from a conceptual standpoint, the subsequent analysis extends these notions to the empirical dataset by evaluating how each stock's total variance is separated into systematic and idiosyncratic parts under the Single Index Model.

Figure 1.4.3 - Systematic and Idiosyncratic Variance Decomposition



Source: author's elaboration using MATLAB outputs.

The figure 1.4.3 presents the variance decomposition for all securities in the sample, ranked in descending order by their overall SIM variance: for each stock, the blue portion denotes the systematic component, whereas the orange portion reflects the idiosyncratic contribution (Elton et al., 2014), several salient empirical regularities can be observed in the graph.

First, the magnitude of total variance varies markedly between securities, evidencing substantial heterogeneity in the risk characteristics of the firms in the sample, in particular way, while some stocks display pronounced volatility, others show comparatively stable patterns in their returns, and such spread is economically plausible given that the dataset draws from a diversified U.S. equity universe, comprising firms from different sectors, industries, and phases of business development; a further key finding relates to the relative contribution of systematic versus idiosyncratic risk: across a substantial part of the sample, the idiosyncratic component accounts for a large fraction of total variance, and in multiple instances it surpasses the systematic portion, indicating that firm processes are central to understanding the volatility of individual equities,

therefore, economically, these firms appear to be more strongly influenced by company-level developments that cannot be fully captured by movements in aggregate market conditions alone (Elton et al., 2014; Herskovic et al., 2016).

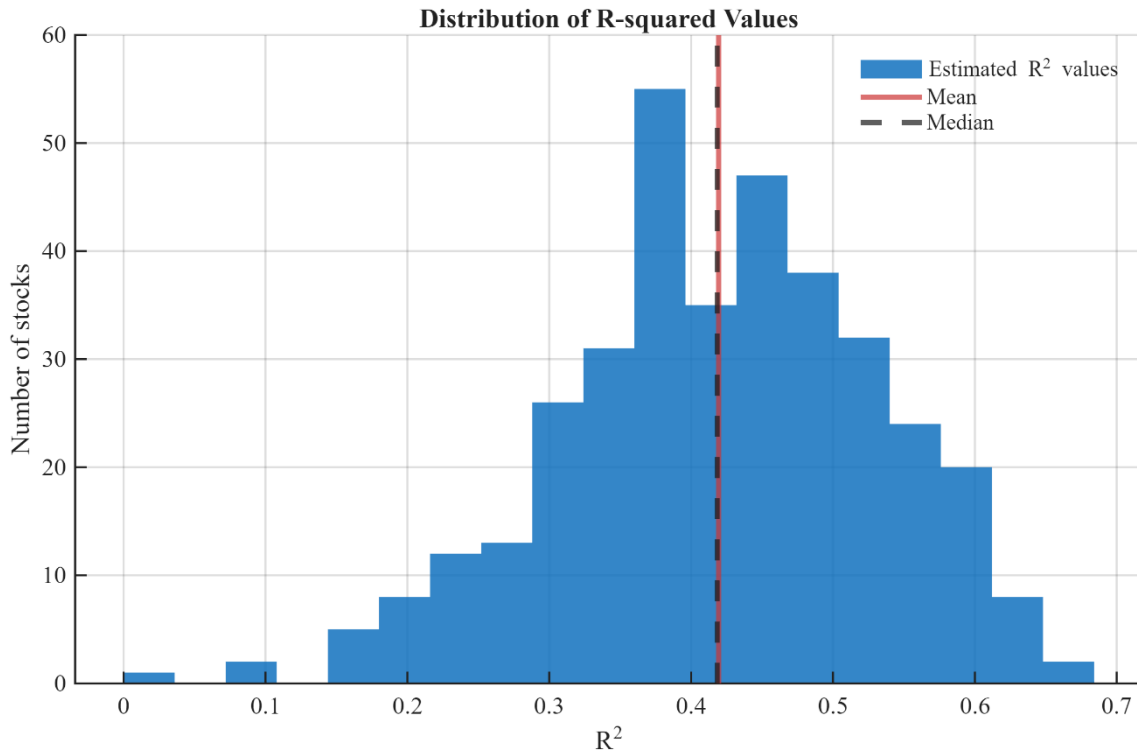
At the same time, the graph indicates that systematic risk is present across essentially all securities, meaning that even where idiosyncratic variance accounts for the majority of total volatility, the market-related component continues to represent a meaningful share, and this result reinforces the core premise of the Single Index Model: although firm-specific shocks are relevant, a common market factor still exerts a broad and pervasive influence on stock returns (Elton et al., 2014; Sharpe, 1964). Another significant feature is the progressive reduction in total variance from left to right in the figure, indeed the lack of sudden discontinuities or isolated extreme peaks indicates that the sample is not governed by highly risky securities, instead risk is distributed fairly smoothly across firms, which supports a more equitable and reliable estimation basis for the subsequent portfolio optimization analyses.

Overall, the variance decomposition suggests that portfolio risk stems from two sources: firms' common exposure to overall market movements and differences in firms' individual risk levels; diversification therefore reduces much of the volatility by offsetting idiosyncratic fluctuations, while risk related to market-wide factors cannot be diversified away and remains non-diversifiable (Markowitz, 1959).

#### 1.4.3 Distribution of $R^2$

In the preceding section, the coefficient of determination,  $R^2$ , was presented as an indicator of the Single Index Model's explanatory capability, here the subsequent analysis extends this framework to the empirical dataset by investigating how the estimated  $R^2$  values are distributed among the stocks included in the sample.

Figure 1.4.4 - Distribution of Estimated  $R^2$  Values



Source: author's elaboration using MATLAB outputs.

Figure 1.4.4 shows that values are largely concentrated at intermediate levels, with both the mean and the median clustering around 0.4, meaning that the market factor explains a relevant portion of the variability in total returns, although it does not fully account for the overall explanatory power of the model.

Confirming the heterogeneity of the sample, also in this case observing the graph we notice a pronounced dispersion: a subset of securities records comparatively high  $R^2$  values, pointing to a strong correspondence between their return behaviour and general market trends, in contrast, a notable share of the sample displays lower  $R^2$  levels, suggesting that idiosyncratic factors are more influential, including shocks specific to the sector, innovation and competitive dynamics, managerial and strategic choices, and other firm-level events that only partially co-move with market returns (Elton et al., 2014; Herskovic et al., 2016). Moreover, it is interesting to note that the distribution is not concentrated at the extremes of 0 or 1, indicating that the dataset is neither dominated by purely market-driven stocks nor by fully idiosyncratic securities.

### 1.5 Residual Correlation Analysis

After demonstrating that the SIM explains a substantial portion of the correlation of stock returns through exposure to the common market factor, it is crucial to explain that the model is based on a restrictive simplifying premise: after eliminating the effects of market movements, the residual components of the different stocks are assumed to be mutually independent (Elton et al., 2014), and this assumption is formulated as follows:

$$E(e_i e_j) = 0 \quad i \neq j$$

or, alternatively, expressed in terms of correlation:

$$\text{Corr}(e_i, e_j) = 0 \quad i \neq j$$

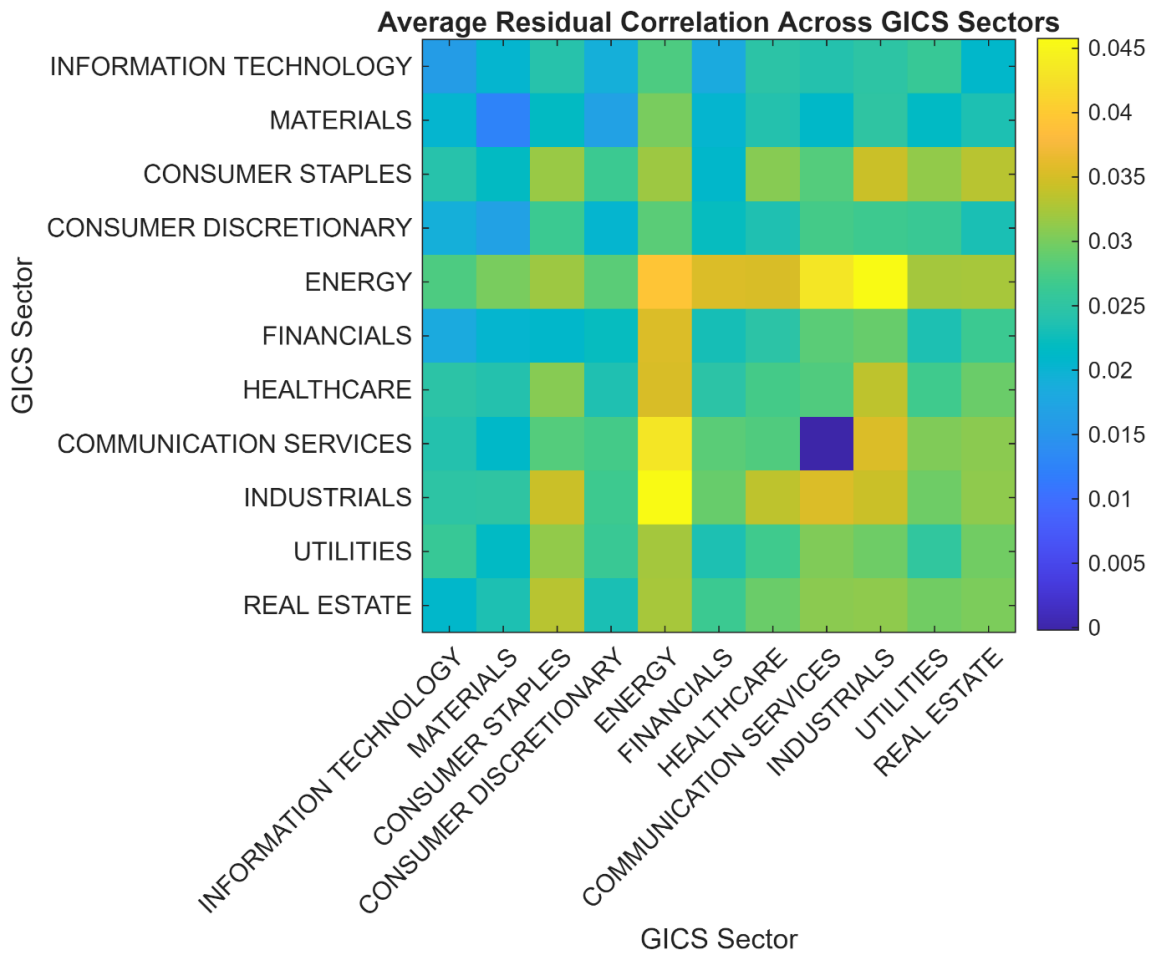
Where:

- $e_i$  identifies the residual component of security  $i$ ;
- $e_j$  denotes the residual component of security  $j$ ;
- $\text{Corr}(e_i, e_j)$  quantifies the correlation between the two residual terms;
- $i \neq j$  meaning that the condition holds for distinct securities (Elton et al., 2014).

So, this assumption implies that once the market factor has been controlled for, there should be no remaining shared source of variation across firms, therefore, the residual returns ought to reflect only idiosyncratic noise specific to each firm (Elton et al., 2014); despite that, the empirical results from this analysis indicate that this assumption is overly restrictive.

To determine whether this assumption is supported empirically in the dataset used for this study, the mean residual correlations across GICS sectors are presented in the figure below.

Figure 1.5.1 - Residual Correlation Structure across GICS Sectors under SIM



Source: author's elaboration using MATLAB outputs.

The heatmap indicates that, even after eliminating the common market component, correlations in the remaining returns do not fully vanish, although these residual correlations are generally substantially lower than those found in the original returns, certain sectors continue to exhibit comparatively higher residual correlation levels than others, which implies that firms within the same industries still move together in ways not completely captured by the Single Index Model, thereby reflecting sector-specific influences in addition to the market factor (Chan et al., 2007).

The most pronounced residual associations tend to cluster within sectors such as Energy, Financials, Industrials, Consumer Staples, and Communication Services; this pattern is economically consistent with the operating characteristics of the firms in these industries: in Energy, residual co-movement likely reflects shared

exposure to oil and gas price dynamics, heightened geopolitical risk, and variations in global demand that influence commodity markets, Financial institutions commonly move in tandem in response to shifts in interest-rate expectations, changes in monetary policy, movements in credit spreads, and liquidity constraints, Industrials are frequently linked through supply-chain interdependencies, recurring capital expenditure cycles, and sensitivity to overall business and production conditions. The heatmap further indicates that consumer-related industries exhibit a comparatively stable residual dependence, this finding implies that firms exposed to household demand may maintain similar responses to inflationary pressures, shifts in consumption, and changes in consumer confidence, even after the aggregate market component has been accounted for and removed (Chan et al., 2007; Bhojraj et al., 2003).

Another relevant insight from the figure is that the residual dependence pattern is not uniform across all sector pairs, meaning that in several cases, the residual correlations are notably weaker, suggesting that the market factor captures a substantial portion of the overall co-movement among securities; at the same time, the continued presence of sector-level clustering implies that the one-factor model still fails to fully remove systematic interdependence across firms.

The conclusion we can draw from this residual correlation analysis indicates that the Single Index Model's assumption of independent residuals may be overly restrictive for a broad and heterogeneous equity universe, such as the one examined in this study, indeed the results obtained support the presence of additional common factors at the sectoral or industry level, whose influence remains detectable even after accounting for market exposure; for this reason, based on this evidence, the analysis will subsequently be extended to the Multi-Index Model, which incorporates multiple systematic components to more fully capture the dependence structure of stock returns (Elton et al., 2014).

## **2 Portfolio Optimization with Mean-CVaR Based on the Single-Index Model**

After outlining the theoretical foundations of the Single-Index Model in the previous chapter, this one addresses the thesis's main goal: designing and optimizing investment portfolios within a Mean-CVaR strategy calibrated on the SIM; specifically, the model serves two complementary functions: first, it offers a compact description of asset-return relationships, which facilitates a more robust and manageable estimation of the covariance structure by compressing pairwise dependence into a market-oriented component and an idiosyncratic residual term (Elton et al., 2014); second, it is used to generate economically coherent return scenarios, which are necessary to perform portfolio optimization under explicitly modelled downside risk (Staino & Russo, 2020; Rockafellar & Uryasev, 2000). By combining factor-based modelling with a tail-risk measure, the proposed approach converts the statistical relationships implied by the Single-Index Model into a risk-sensitive optimization setting; moreover, portfolio weights are chosen to trade off expected performance, reflected by the Mean term, against potential losses in the lower tail of the return distribution, reflected by the CVaR term (Rockafellar & Uryasev, 2000; Strub et al., 2019).

Overall, the framework integrates factor-driven estimation, scenario generation, and downside-risk optimization into a unified methodology, aimed at supporting more resilient portfolio allocation decisions under adverse market conditions (Staino & Russo, 2020; Strub et al., 2019).

To determine the real-world effectiveness of the proposed approach, in this chapter we'll see the development a rolling-window backtesting framework that mirrors a plausible portfolio management process over time: for each month in the sample a six-month estimation period is employed to infer the Single-Index Model parameters (alpha, beta, and the residual variance) for each stock; these estimates are then used to generate hypothetical return paths for the subsequent month; the simulated return scenarios are incorporated into the Mean-CVaR optimization to

obtain portfolio weights that jointly account for expected returns and mitigate downside tail risk; then the computed weights are applied to the realized returns in the following month to measure the portfolio's out-of-sample performance. At month-end, the portfolio is rebalanced and the procedure is repeated, shifting the estimation window forward by one month. In this manner, the backtest assesses the strategy's resilience and performance under time-varying market conditions.

## 2.1 Example month

To illustrate the rolling-window backtesting procedure, the methodology is first demonstrated using a single representative month: this choice enables a close examination of each stage of the full optimization process under the Mean–CVaR (Staino & Russo, 2020; Ferreira et al., 2021).

The selected month is April 2010, as it captures a market recovery phase following the Global Financial Crisis; focusing on this specific time window allows us to replicate the practical sequence that underlies portfolio construction: estimation of the SIM parameters from the information available up to the start of the selected month, generation of future scenarios consistent with those estimates, and determination of the optimal portfolio weights by solving the optimization problem under the Mean–CVaR criterion (Rockafellar & Uryasev, 2000); this method improves the transparency of the framework: rather than relying solely on aggregate backtest statistics to assess whether the model produces value over time, the monthly example clarifies how statistical inputs are mapped into investment decisions at a given point in the sample. This approach, in addition, addresses an important issue emphasized in the empirical literature, namely the sensitivity of optimized strategies to estimation error (Michaud & Michaud, 2007): by observing the transformation of parameter estimates derived from a limited information window into CVaR-aware allocations, the analysis provides a structured and empirically grounded view of how estimation uncertainty affects portfolio weights (Rockafellar & Uryasev, 2000).

### 2.1.1 Estimation Window and SIM estimation

It is essential to specify the historical estimation range used to estimate the model parameters; consequently, in line with the rolling window approach described above, the April 2010 portfolio is constructed exclusively based on information available before the start of the investment period: therefore, the estimation window extends for six months, from October 2009 to March 2010.

Adopting a six-month rolling window offers a pragmatic balance between incorporating the most recent market information and retaining an adequate number of daily observations for robust parameter estimation; furthermore, limiting the analysis to historical data eliminates look-ahead bias and aligns the methodology with a realistic investment decision-making framework (Staino & Russo, 2020).

After the estimation window has been specified, the SIM is estimated separately for each stock in the sample, but since the model's theoretical foundations and interpretative role have been covered in depth in Chapter 2, this section addresses only the model's practical use within the portfolio optimization setting; therefore, the estimated parameters are then used to construct simulated return scenarios for the subsequent month, which serve as the inputs for the Mean–CVaR optimization procedure (Elton et al., 2014; Rockafellar & Uryasev, 2000).

*Figure 2.1.1 - Top 10 Stocks with the Highest Estimated Beta Coefficients during the October 2009 – March 2010 Estimation Window*

| Stock                                       | GICS Sector            | Alpha   | Beta   | SigmaEps | R2     | MuSIM  | VarSIM |
|---------------------------------------------|------------------------|---------|--------|----------|--------|--------|--------|
| APA Corporation (NasdaqGS:APA)              | ENERGY                 | -0,0008 | 2,3970 | 0,0321   | 0,3245 | 0,0015 | 0,0015 |
| O'Reilly Automotive, Inc. (NasdaqGS:ORLY)   | CONSUMER DISCRETIONARY | 0,0007  | 2,2166 | 0,0245   | 0,4131 | 0,0028 | 0,0010 |
| Simon Property Group, Inc. (NYSE:SPG)       | REAL ESTATE            | 0,0009  | 2,1959 | 0,0185   | 0,5490 | 0,0030 | 0,0008 |
| Viatris Inc. (NasdaqGS:VTRS)                | HEALTHCARE             | 0,0004  | 2,1711 | 0,0173   | 0,5767 | 0,0025 | 0,0007 |
| Kimberly-Clark Corporation (NYSE:KMB)       | CONSUMER STAPLES       | 0,0000  | 2,0623 | 0,0170   | 0,5601 | 0,0020 | 0,0007 |
| Arch Capital Group Ltd. (NasdaqGS:ACGL)     | FINANCIALS             | -0,0015 | 1,9789 | 0,0209   | 0,4349 | 0,0004 | 0,0008 |
| Revvity, Inc. (NYSE:RVTY)                   | HEALTHCARE             | 0,0000  | 1,9634 | 0,0177   | 0,5154 | 0,0019 | 0,0006 |
| The Estée Lauder Companies Inc. (NYSE:EL)   | CONSUMER STAPLES       | 0,0000  | 1,9403 | 0,0210   | 0,4238 | 0,0019 | 0,0008 |
| Consolidated Edison, Inc. (NYSE:ED)         | UTILITIES              | -0,0013 | 1,8827 | 0,0242   | 0,3421 | 0,0005 | 0,0009 |
| Air Products and Chemicals, Inc. (NYSE:APD) | MATERIALS              | 0,0000  | 1,8719 | 0,0200   | 0,4311 | 0,0018 | 0,0007 |

Source: author's elaboration using MATLAB outputs.

The table above presents the ten stocks with the highest estimated beta coefficients over the chosen estimation window: ranking the securities by beta enables us to determine which firms display the greatest responsiveness to market fluctuations during the months prior to April 2010 (Elton et al., 2014). Looking at the table what can be seen is that the largest beta estimates are observed for companies in

sectors including Energy, Financials, Real Estate, Materials, and Consumer Discretionary, so industries that are generally more exposed to shifts in macroeconomic conditions and changes in investor sentiment (Chan et al., 2007). In particular, APA Corporation and O'Reilly Automotive show the highest estimated betas in the sample, with values greater than 2, suggesting substantial sensitivity to market-wide movements during the estimation period; similarly, Simon Property Group and Arch Capital Group indicate that Real Estate and Financial stocks respond strongly to changes in broader market conditions in the post-crisis recovery stage.

The table further suggests that the model's explanatory ability differs moderately across individual securities, as reflected in the  $R^2$  values observed in the table, meaning that for several stocks, a substantial portion of return variability is still linked to firm-specific influences, indicating that even during periods of market rebound, idiosyncratic company factors continue to coexist with broader market dynamics. Finally, the inferred estimates of MuSIM and VarSIM yield the expected return and variance implied by the model for each security, in particular way while securities with the largest betas generally correspond to higher expected returns in the model, they likewise display greater levels of estimated risk, in line with their stronger exposure to market fluctuations (Elton et al., 2014).

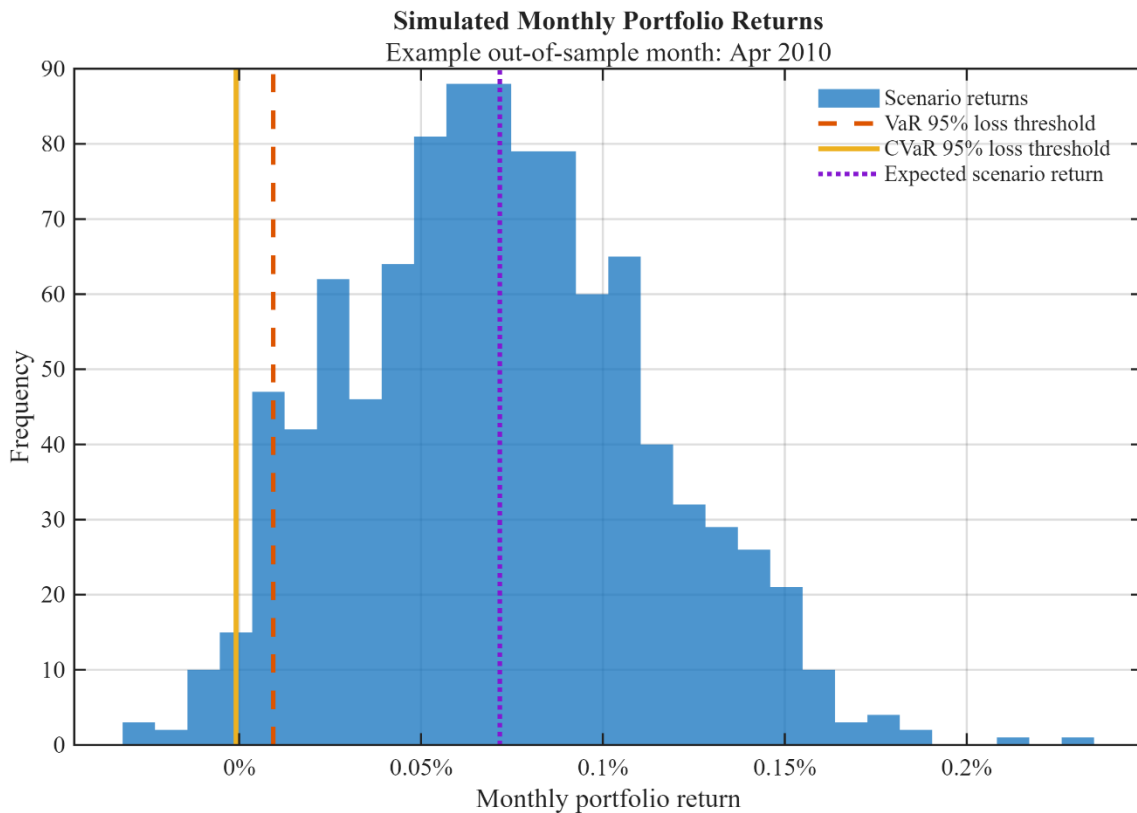
Overall, this stage converts the historical information within the estimation window into the numerical inputs needed to support the subsequent stages of scenario generation and Mean-CVaR portfolio optimization.

### 2.1.2 Scenario generation

The following phase involves generating a large collection of plausible return scenarios for the out-of-sample period corresponding to April 2010; instead of producing a single point forecast, this approach aims to develop a probabilistic description of the range of market states that could likely emerge during the subsequent investment horizon (Staino & Russo, 2020).

The scenario generation procedure relies on the parameters estimated during the October 2009–March 2010 period: the simulation starts from the market factor, for which expected return and volatility are derived from the historical dataset; then, for each simulated realization of the market factor, the corresponding stock returns are produced consistently with the sensitivities estimated under the SIM (Ponomareva et al., 2015; Staino & Russo, 2020). In the empirical application, 1,000 monthly scenarios are generated, allowing the optimization routine to evaluate the expected portfolio return and the range of potential outcomes (Ferreira et al., 2021); a key feature of this approach is that it preserves the economic dependencies implicit in the SIM, while generating variation across simulated paths; consequently, the optimization procedure can reflect both favourable and unfavourable market scenarios, with particular attention to the lower tail of the return distribution, an essential element for the Mean-CVaR methodology (Rockafeller & Uryasev, 2000).

Figure 2.1.2 - Histogram of Simulated Monthly Portfolio Returns for the April 2010 Out-of-Sample Period



Source: author's elaboration using MATLAB outputs.

Figure 2.1.2 presents the histogram of the simulated monthly portfolio returns for the April 2010 out-of-sample period, from which we can deduce that the resulting distribution is largely concentrated in the positive return range, indicating that most simulated outcomes correspond to moderately positive monthly returns; therefore, based on the information available at the end of March 2010, the simulation suggests a higher probability of observing positive portfolio outcomes in the coming month.

The dotted purple line indicates the expected portfolio return across the simulated scenarios: its location close to the centre of the distribution shows that the mean simulated outcome is positive, in line with the market recovery documented in the post-crisis period (Ferreira et al., 2021); the dashed orange curve denotes the 95% VaR level, meaning the return threshold exceeded only by the most adverse 5% of the simulated outcomes; while the solid yellow curve corresponds to the 95% CVaR, which captures the mean loss conditional on these extreme tail events, as expected, the CVaR threshold is positioned further to the left than the VaR threshold, since losses averaged over the tail are more severe than the loss value defined solely by the percentile cutoff (Rockafellar & Uryasev, 2000).

The simulated return distribution serves as the crucial input for the subsequent optimization problem, since the final interpretation of the graph implies that even if the overall distribution is tilted toward positive outcomes, the left tail remains substantial, which is a relevant point in our Mean-Cvar optimization setting: portfolio construction beyond enhancing expected returns aims also at limiting the size of potential extreme losses (Rockafellar & Uryasev, 2000).

### 2.1.3 Mean-CVaR optimization problem

The subsequent task is to identify the optimal portfolio weights using a Mean–CVaR approach, although the discussion draws on the April 2010 illustrative month, the procedure that follows sets out the Mean–CVaR optimization methodology applied throughout the entire backtesting exercise; this step aims to determine the weight vector by jointly accounting for the return levels obtained by

the simulations and the degree of exposure to downside tail risk (Rockafellar & Uryasev, 2000); In the empirical exercise, the simulated returns produced by the Single-Index Model are arranged in the scenario matrix  $R_{scen}$  (Fairbrother et al., 2022): each row of  $R_{scen}$  represents a distinct simulated monthly scenario, whereas each column corresponds to a particular stock within the investment universe, in particular way, given a portfolio weight vector  $w$  the return of the portfolio under scenario  $s$  is defined as:

$$r_s(w) = R_{scen,s}w$$

where  $R_{scen,s}$  denotes the  $s$ -th row of the scenario matrix. The expected portfolio return over the simulated scenarios is then obtained as:

$$\mu_p = \frac{1}{S} \sum_{s=1}^S r_s(w)$$

where  $S$  denotes the total number of simulated scenarios; in this context, the expression corresponds to the “Mean” term of the optimization objective, reflecting the average expected portfolio return over the simulated return distribution (Rockafellar & Uryasev, 2000).

Concerning the risk component, the analysis sets the confidence level at  $\beta = 0.95$ , in line with the MATLAB implementation: at this percentile, VaR determines the loss threshold corresponding to the most adverse 5% of the simulated outcomes, whereas CVaR computes the mean loss conditional on losses exceeding that threshold, therefore CVaR captures both the onset of the left tail and the magnitude of losses occurring within it (Rockafellar & Uryasev, 2000).

Building on Rockafellar and Uryasev (2000), CVaR minimization can be expressed as a computationally efficient linear programming model by adding suitable auxiliary variables; in a scenario-based representation, the optimization problem is formulated as:

$$\min \alpha + \frac{1}{S(1-\beta)} \sum_{s=1}^S u_s$$

here,  $\alpha$  denotes the VaR threshold, while  $u_s$  captures the excess loss under scenario

s, this specification is particularly advantageous since it enables the downside-risk minimization problem to be addressed efficiently using linear programming (Rockafellar & Uryasev, 2000).

In the implemented MATLAB code, the decision variables comprise the portfolio weights  $w_i$ , the threshold parameter  $\alpha$ , and the slack variable  $u_s$ , then the optimization problem is addressed via the linprog function under the following set of constraints (Ferreira et al., 2021).

1. Financial constraint

$$\sum_{i=1}^N w_i = 1$$

This condition entails the full allocation of the available capital across the relevant investment or operational channels, leaving no residual funds uncommitted.

2. Long-Only Limitation

$$w_i \geq 0$$

This constraint rules out short-selling and ensures that the portfolio consists solely of long positions: as a result, the investment approach depends exclusively on profiting from upward price movements, rather than on any potential returns associated with falling asset prices.

3. Minimum required expected return condition

$$\mu'w \geq \text{targetReturn}$$

This constraint integrates the return component into the optimization problem and reduces the likelihood of an excessively cautious portfolio allocation, meaning that in the implemented algorithm, the target return is not specified externally as a fixed parameter but it is computed adaptively based on the simulated scenario distribution generated for each rebalancing month.

For the example month of April 2010, the target return is derived as a fraction of the equally weighted average scenario return, as implied by the simulated distribution, in this case, because the simulated setting is associated with favourable expected market conditions, the optimization framework calibrates the

minimum required expected return to be 80% of the equally weighted average scenario return; this specification is designed to achieve a realistic compromise between the expected performance of the portfolio and the containment of downside risk: by setting the threshold slightly below the mean simulated return, the optimization problem is prevented from becoming overly aggressive, while the portfolio remains aligned with a substantive expected return target; in addition, the adaptive nature of this specification supports greater numerical stability and enhances the feasibility of the optimization procedure under varying market conditions.

#### 4. Constraints for the linearization of CVar

$$u_s \geq -r_s(w) - \alpha, u_s \geq 0$$

for each scenario  $s = 1, \dots, S$ . These constraints separate the losses whose values lie beyond the VaR limit, thereby enabling the minimization of their mean magnitude within a linear optimization framework (Rockafellar & Uryasev, 2000; Ferreira et al., 2021).

For the April 2010 example month, this optimization is applied to the 1,000 simulated monthly return scenarios generated in the preceding step, this stage constitutes the core of the empirical procedure: the Single-Index Model yields the simulated return distribution, whereas the Mean-CVaR optimization converts this distribution into a specific portfolio allocation (Rockafellar & Uryasev, 2000; Elton et al., 2014) the result of this phase is the vector of optimal weights, which is examined in the subsequent section.

##### 2.1.4 Portfolio weights

After solving the Mean-CVaR optimization problem, the procedure yields the vector of optimal portfolio weights  $w^*$ , which specifies how the available capital is distributed among the selected securities for the April 2010 reference period (Rockafellar & Uryasev, 2000).

$$w^* = (w_1, w_2, \dots, w_N)$$

The allocation obtained reflects the operational impact of the entire framework

established in the preceding stages: the information drawn from the estimation window and embedded in the simulated return scenarios is converted into a concrete investment choice via the optimization procedure; therefore the optimizer allocates higher weights to those securities that, in the simulated distribution, deliver a more advantageous balance between expected returns and exposure to downside risk (Staino & Russo, 2020; Rockafellar & Uryasev, 2000).

*Figure 2.1.3 - Optimal Portfolio Weights for the April 2010 Out-of-Sample Month*

| Stock                                                    | GICS Sector            | Alpha  | Beta   | MuSIM  | VarSIM | Weight        |
|----------------------------------------------------------|------------------------|--------|--------|--------|--------|---------------|
| The Clorox Company (NYSE:CLX)                            | CONSUMER STAPLES       | 0,0030 | 0,6103 | 0,0036 | 0,0003 | <b>0,1528</b> |
| Masco Corporation (NYSE:MAS)                             | INDUSTRIALS            | 0,0039 | 0,6405 | 0,0045 | 0,0003 | <b>0,1472</b> |
| Autodesk, Inc. (NasdaqGS:ADSK)                           | INFORMATION TECHNOLOGY | 0,0034 | 0,2184 | 0,0036 | 0,0009 | <b>0,0945</b> |
| NextEra Energy, Inc. (NYSE:NEE)                          | UTILITIES              | 0,0027 | 0,9482 | 0,0036 | 0,0006 | <b>0,0746</b> |
| eBay Inc. (NasdaqGS:EBAY)                                | CONSUMER DISCRETIONARY | 0,0022 | 0,7699 | 0,0030 | 0,0002 | <b>0,0700</b> |
| Norfolk Southern Corporation (NYSE:NSC)                  | INDUSTRIALS            | 0,0015 | 0,7322 | 0,0022 | 0,0001 | <b>0,0621</b> |
| Regeneron Pharmaceuticals, Inc. (NasdaqGS:REGN)          | HEALTHCARE             | 0,0011 | 0,4006 | 0,0015 | 0,0001 | <b>0,0536</b> |
| The Campbell's Company (NasdaqGS:CPB)                    | CONSUMER STAPLES       | 0,0022 | 0,6244 | 0,0028 | 0,0003 | <b>0,0460</b> |
| Stanley Black & Decker, Inc. (NYSE:SWK)                  | INDUSTRIALS            | 0,0020 | 0,9833 | 0,0029 | 0,0003 | <b>0,0459</b> |
| Cisco Systems, Inc. (NasdaqGS:CSCO)                      | INFORMATION TECHNOLOGY | 0,0009 | 0,5039 | 0,0014 | 0,0001 | <b>0,0447</b> |
| QUALCOMM Incorporated (NasdaqGS:QCOM)                    | INFORMATION TECHNOLOGY | 0,0017 | 0,9657 | 0,0026 | 0,0002 | <b>0,0382</b> |
| CarMax, Inc. (NYSE:KMX)                                  | CONSUMER DISCRETIONARY | 0,0029 | 1,5031 | 0,0044 | 0,0004 | <b>0,0356</b> |
| Everest Group, Ltd. (NYSE:EG)                            | FINANCIALS             | 0,0015 | 0,6828 | 0,0022 | 0,0003 | <b>0,0353</b> |
| Expeditors International of Washington, Inc. (NYSE:EXPD) | INDUSTRIALS            | 0,0035 | 1,3552 | 0,0048 | 0,0006 | <b>0,0298</b> |
| M&T Bank Corporation (NYSE:MTB)                          | FINANCIALS             | 0,0010 | 0,5755 | 0,0015 | 0,0001 | <b>0,0252</b> |
| Jacobs Solutions Inc. (NYSE:J)                           | INDUSTRIALS            | 0,0027 | 1,4407 | 0,0041 | 0,0006 | <b>0,0161</b> |
| Snap-on Incorporated (NYSE:SNA)                          | INDUSTRIALS            | 0,0024 | 0,8486 | 0,0032 | 0,0004 | <b>0,0111</b> |
| UDR, Inc. (NYSE:UDR)                                     | REAL ESTATE            | 0,0008 | 0,6531 | 0,0014 | 0,0001 | <b>0,0088</b> |
| General Mills, Inc. (NYSE:GIS)                           | CONSUMER STAPLES       | 0,0017 | 0,7008 | 0,0024 | 0,0002 | <b>0,0073</b> |
| Eastman Chemical Company (NYSE:EMN)                      | MATERIALS              | 0,0010 | 0,4838 | 0,0014 | 0,0001 | <b>0,0011</b> |

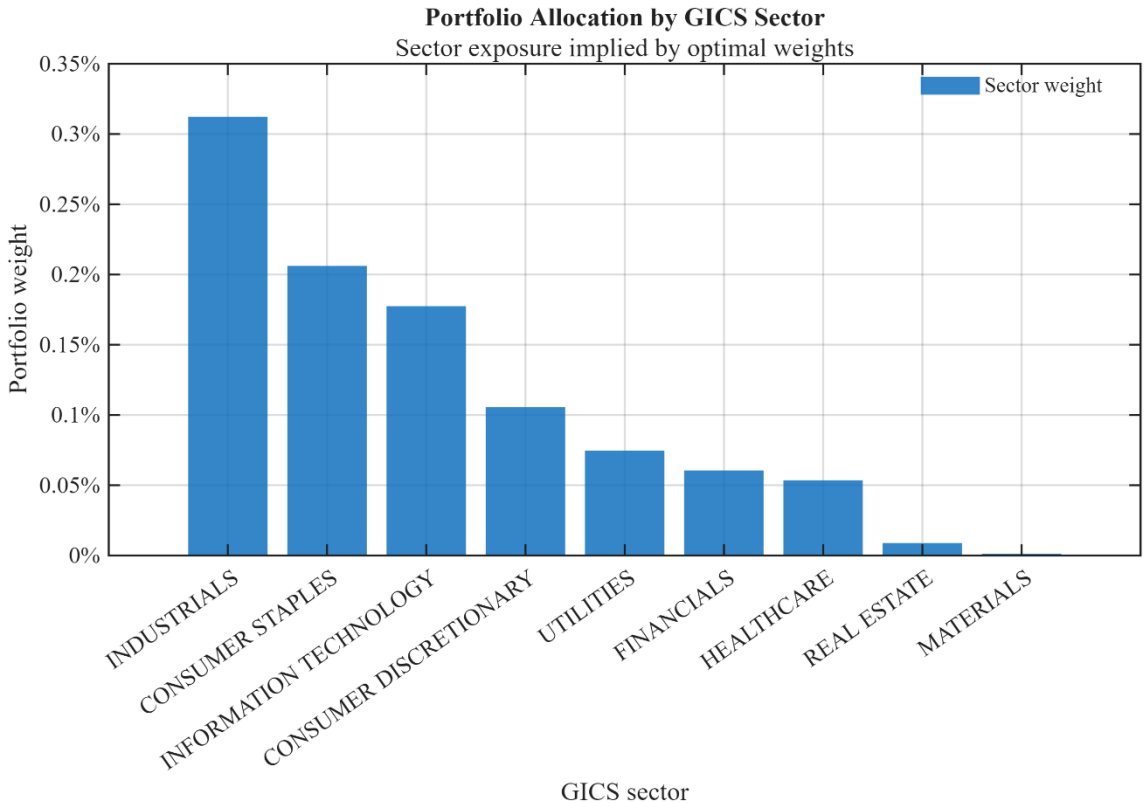
*Source: author's elaboration using MATLAB outputs.*

The final allocation is presented in the weights table, which lists the selected securities in descending order based on their portfolio weights: for each security, the table provides the GICS sector classification, the corresponding estimated SIM statistics, and the portfolio exposure ultimately assigned by the optimizer (Chan et al., 2007; Elton et al., 2014).

For April 2010, the portfolio allocation is moderately concentrated, with the highest weights assigned to The Clorox Company, Masco Corporation, Autodesk, NextEra Energy, and eBay, as a consequence, what is deduced is that several of the most heavily weighted securities are drawn from Industrials, Consumer Staples, and Information Technology, suggesting that the optimization framework regards these sectors as comparatively favourable within the simulated market conditions characterizing the post-crisis recovery period (Staino & Russo, 2020). At the same time, the allocation is not overly concentrated in only a handful of

assets: the existence of many smaller positions indicates that the optimizer continues to embed a measure of diversification across different firms and sectors (Markowitz, 1952; Elton et al., 2014); furthermore, the portfolio composition indicates that larger allocations are not necessarily associated with the highest beta values, because the optimizer evaluates the entire simulated return distribution, simultaneously incorporating expected returns, downside-tail behaviour, and cross-asset dependencies embedded in the scenario framework, assets with relatively limited sensitivity to market movements may nonetheless receive significant weights when they enhance the portfolio’s overall risk–return balance. (Rockafellar & Uryasev, 2000; Fairbrother et al., 2022).

Figure 2.1.4 - Portfolio Allocation by GICS Sector for the April 2010 Out-of-Sample Month



Source: author’s elaboration using MATLAB outputs.

To better understand, the sector-level allocation displayed in the figure above offers a more comprehensive interpretation of the optimized portfolio configuration: by consolidating the individual stock weights using the GICS

classification, the figure enables an assessment of whether the final allocation is concentrated within particular segments of the market or broadly distributed across multiple sources of exposure (Chan et al., 2007; Bhojraj et al., 2003).

For the April 2010 reference month, the highest weight is allocated to Industrials, accounting for slightly more than 30% of the overall portfolio, this outcome is primarily associated with the inclusion of multiple industrial firms among the selected securities, namely Masco Corporation, Norfolk Southern, Stanley Black & Decker, Jacobs Solutions, and Snap-on; So the prominence of this sector indicates that, across the simulated scenarios, industrial equities provided a favourable balance between expected returns and limited exposure to downside risk (Rockafellar & Uryasev, 2000; Elton et al., 2014).

The second largest allocation is directed toward Consumer Staples, with Information Technology following closely: this arrangement is noteworthy because it reflects the integration of two distinct exposure profiles: Consumer Staples adds a defensive dimension (demand tends to remain relatively stable over time), illustrated by firms such as Clorox, Campbell's and General Mills, whereas Information Technology represents the portfolio through companies including Autodesk, Cisco and Qualcomm, this sector is generally more exposed to the economic cycle and has higher growth prospects; Consumer Discretionary also receives a substantial allocation, chiefly via eBay and CarMax, which suggests that the optimization procedure assigns some emphasis to sectors closely associated with consumer activity and economic rebound; by contrast, Utilities, Financials, and Healthcare are weighted more moderately, whereas Real Estate and Materials retain comparatively minor influence.

Overall, the GICS allocation indicates that the portfolio is diversified across multiple sectors, then the final composition is skewed toward Industrials, Consumer Staples, and Information Technology, whose combined weight represents the largest share of the allocation.

### 2.1.5 Out-of-Sample Realized Return

The next step involves testing the resulting portfolio using the market data actually observed in the following month, a procedure that provides the true out-of-sample evaluation, because the realized returns employed for performance measurement were not used during either the estimation stage or the generation of the return scenarios (Staino & Russo, 2020).

Concretely, the optimized weight vector  $w^*$  is applied to the realized daily returns of the chosen securities over April 2010, then the corresponding daily portfolio return is computed as:

$$r_{p,t}^{real} = R_{real,t} w^*$$

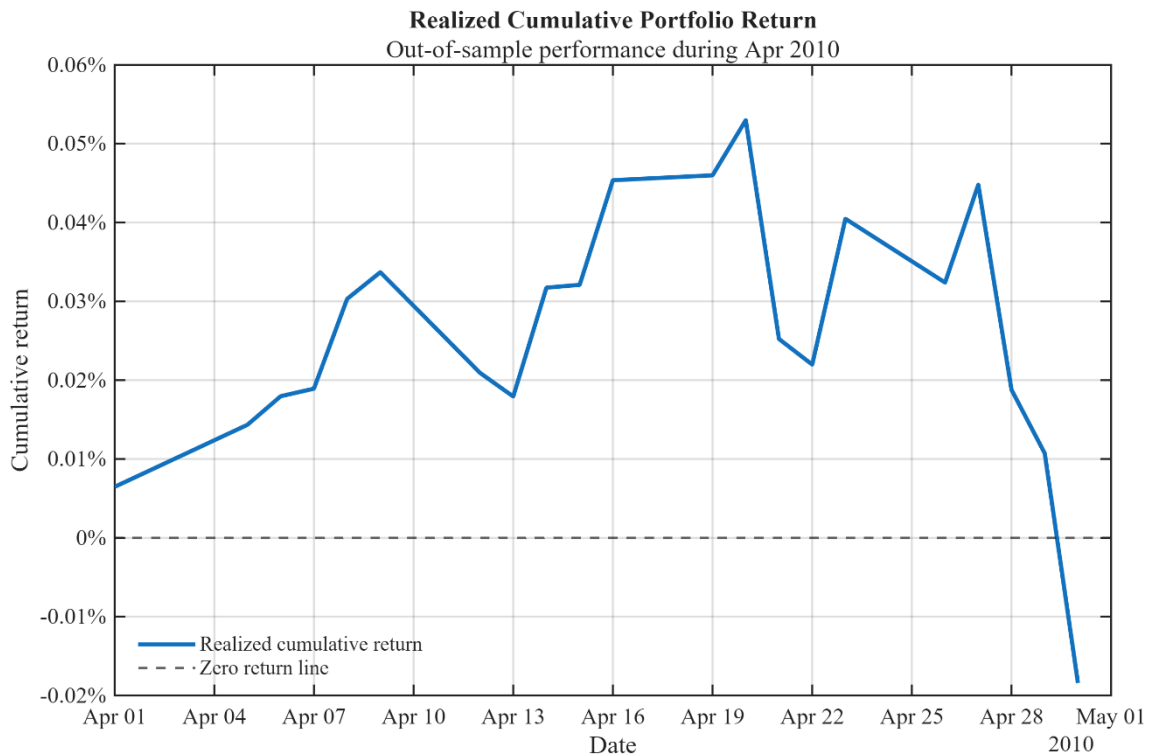
Where  $R_{real,t}$  indicates the vector of observed asset returns on trading day  $t$ , whereas  $w^*$  denotes the portfolio weights set and kept fixed at the conclusion of the optimization phase.

The realized monthly portfolio return is subsequently derived by geometrically compounding the daily portfolio returns:

$$R_p^{real} = \prod_{t=1}^D (1 + r_{p,t}^{real}) - 1$$

where  $D$  indicates the number of trading days (21 in that case) in the out-of-sample month. This formulation reflects the cumulative effect of sequential gains and losses over time and is therefore coherent with practical portfolio performance measurement (Elton et al., 2014).

Figure 2.1.5 - Realized Cumulative Portfolio Return during the April 2010 Out-of-Sample Month



Source: author's elaboration using MATLAB outputs.

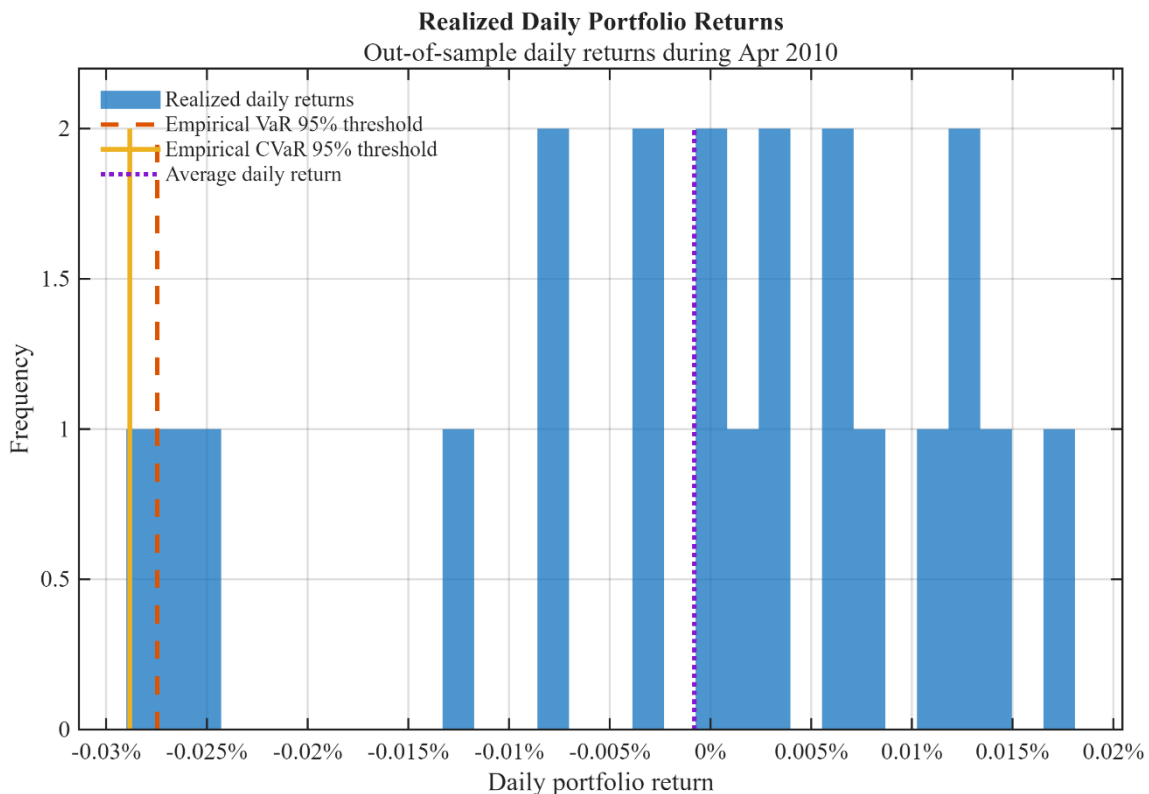
Figure 2.1.5 shows the cumulative daily performance of the optimized portfolio over April 2010. Overall, the portfolio performed consistently well, remaining above the zero-return reference for most of the month, cumulative returns increased steadily during the first half of April and reached a local peak of slightly above 5% around the third week of trading, this performance confirms that the portfolio allocation benefited from the generally favourable market conditions characterising the post-crisis recovery period.

Final trading periods, however, show a clear deterioration in performance, although the cumulative return remains positive for most of the investment horizon, it declines sharply toward the end of the month and eventually falls slightly below zero, reaching approximately  $-1.8\%$ ; this result indicates that, even when a portfolio is constructed under optimistic simulated assumptions, it may still be exposed to sudden adverse market movements during the evaluation period.

Figure 2.1.5 from an economic perspective highlights the distinction between ex ante portfolio optimization and ex post outcomes: the allocation weights were

determined exclusively on the basis of the information available at the end of March 2010 and were optimal given the simulated return distribution generated by the model; nevertheless, the realized results depend on the specific sequence of returns observed over the subsequent month, which can diverge markedly from the central reference value implicit in the scenarios (Staino & Russo, 2020).

*Figure 2.1.6 - Distribution of Realized Daily Portfolio Returns during the April 2010 Out-of-Sample Month*



*Source: author's elaboration using MATLAB outputs.*

To support this, figure 2.1.6 presents the empirical distribution of the portfolio's daily returns during April 2010: the histogram indicates that the majority of daily observations cluster around modest positive values or returns close to zero, in line with the generally positive performance observed for much of the month; but it's also important to notice a small number of substantially negative daily outcomes that play a decisive role in producing the month's net reversal. Also in this graph we can observe the position of the dashed orange curve that corresponds to the empirical 95% VaR threshold, while the solid yellow line represents the empirical

95% CVaR threshold, instead the dotted purple line instead indicates the mean realized daily return; the extent to which the empirical left tail lies below the average return corroborates that downside episodes exist even in months where overall market conditions appear broadly favourable (Staino & Russo, 2020).

The April 2010 example highlights an important feature of rolling portfolio optimization frameworks: although the simulated scenarios and the resulting allocation may initially seem economically attractive, actual performance can still be significantly affected by short-term market volatility during the investment period and may diverge substantially from ex ante expectations under adverse market conditions (Staino & Russo, 2020).

#### 2.1.6 Interpretation and conclusions

*Table 2.1.1 - Summary Statistics for the April 2010 Example Month*

|                                    |            |
|------------------------------------|------------|
| <b>Estimation Start</b>            | 01/10/2009 |
| <b>Estimation End</b>              | 31/03/2010 |
| <b>Test Start</b>                  | 01/04/2010 |
| <b>Test End</b>                    | 30/04/2010 |
| <b>Number of Assets</b>            | 359        |
| <b>Active Positions</b>            | 20         |
| <b>Number of Trading Days Test</b> | 21         |
| <b>Number of Scenarios</b>         | 1000       |
| <b>MuMarket</b>                    | 0,0958%    |
| <b>SdMarket</b>                    | 0,9286%    |
| <b>Target Return</b>               | 2,3733%    |
| <b>AlphaP</b>                      | 0,2530%    |
| <b>BetaP</b>                       | 0,70965    |
| <b>MuP SIM</b>                     | 0,3210%    |
| <b>VarP SIM</b>                    | 0,00007    |
| <b>Scenarios Expected Return</b>   | 7,1640%    |
| <b>Scenario VaR</b>                | 0,9396%    |
| <b>Scenario CVaR</b>               | -0,0663%   |
| <b>Realized Return</b>             | -1,8412%   |

*Source: author's elaboration based on MATLAB outputs.*

Table 2.1.1 presents a synthetic summary of the principal outcomes obtained for the April 2010 example month and provides the foundation for the subsequent

interpretations and final remarks concerning the estimation, simulation, optimization, and out-of-sample evaluation phases.

The portfolio is formed using a six-month estimation period from October 2009 to March 2010, whereas the assessment is carried out in April 2010, instead the asset universe comprises 359 securities, yet the optimized solution allocates weights to only 20 active positions: this outcome shows that the Mean–CVaR optimization approach leads to a highly selective allocation strategy (Ferreira et al., 2021; Rockafellar & Uryasev, 2000).

The simulated market conditions derived from the Single-Index Model are moderately positive on the whole: specifically, the optimized portfolio exhibits a positive expected return of approximately 7.16%, while its beta remains below unity ( $\beta P=0.70965$ ), implying systematic risk exposure lower than that of the market portfolio; moreover, the estimated portfolio alpha is positive, suggesting that the allocation is not driven solely by market-related exposure (Elton et al., 2014).

From a downside-risk perspective, the simulated tail-risk measures are consistent with the positively skewed return distribution identified in the scenario analysis: the scenario VaR is approximately 0.94%, indicating that only 5% of the simulated outcomes generate portfolio returns below this threshold, the corresponding scenario CVaR is around  $-0.066\%$ , suggesting that average losses remain relatively limited even under the most adverse simulated conditions; these findings demonstrate that, given the assumed market conditions, the optimized portfolio provides an advantageous trade-off between return generation and the mitigation of downside risk. (Rockafellar & Uryasev, 2000).

Nevertheless, the realized results diverge substantially from the references implied by the simulated scenarios: while cumulative returns remain positive for much of April 2010, performance deteriorates markedly in the final trading sessions, resulting in a realized monthly return of approximately  $-1.84\%$ . This gap between simulated expectations and actual outcomes underscores a key feature of financial markets: notwithstanding the use of consistent statistical and risk-based

optimization procedures, realized returns are still exposed to unforeseen short-term volatility and market shocks (Fairbrother et al., 2022; Staino & Russo, 2020).

In conclusion, the April 2010 illustration clearly explains the economic reasoning behind the proposed approach: initially, historical data are converted into estimates of the SIM parameters, these estimates are then employed to construct forward-looking return scenarios, finally, the scenarios are introduced into a Mean–CVaR optimization problem to identify the optimal portfolio weights; the subsequent out-of-sample evaluation offers a concrete basis for judging the strategy’s performance under real market conditions (Ponomareva et al., 2015; Rockafellar & Uryasev, 2000).

Accordingly, the rationale for using an illustrative month is not to judge the methodology’s performance based on a single observed return, rather, it is intended to transparently demonstrate the complete rolling backtesting procedure, which will subsequently be applied to the full-sample analysis.

## 2.2 Rolling Backtest of the SIM Mean–CVaR Portfolio Strategy

After demonstrating the full optimization procedure with the April 2010 case, the same methodology can be applied to the entire observation window (December 2007 to March 2021) through a rolling-window backtesting approach.

While the earlier section was mainly intended to make clear the mechanics of the methodology (namely, parameter estimation, scenario construction, portfolio optimization, and performance assessment) by focusing on a single month, the analysis that follows, instead, concentrates on how the strategy evolves over time (Rockafellar & Uryasev, 2000): rolling-window backtesting enables us to gauge whether the SIM-based Mean-CVaR approach continues to perform consistently across diverse market conditions, including episodes marked by elevated volatility, regime instability, and sudden changes in downside risk.

Through this mechanism it is possible to evaluate the strategy's stability and persistence over the full investment horizon by repeatedly estimating the model parameters, producing updated return scenarios, and rebalancing the portfolio on a monthly schedule while relying solely on information that would have been available at each evaluation date; this methodology offers a more faithful representation of an implementable investment process and shifts the analysis from a single static example to an in-depth evaluation of portfolio performance, risk characteristics, allocation behaviour, and responsiveness to evolving market conditions.

A crucial consideration is the elimination of look-ahead bias: since in the rolling process, portfolio weights are computed solely from the information available at the relevant decision date and the realized returns are assessed only during the following out-of-sample month, such explicit separation between the estimation stage and the evaluation stage sustains the empirical validity of the framework and ensures that information from future market conditions does not affect portfolio construction decisions (Staino & Russo, 2020; Ferreira et al., 2021).

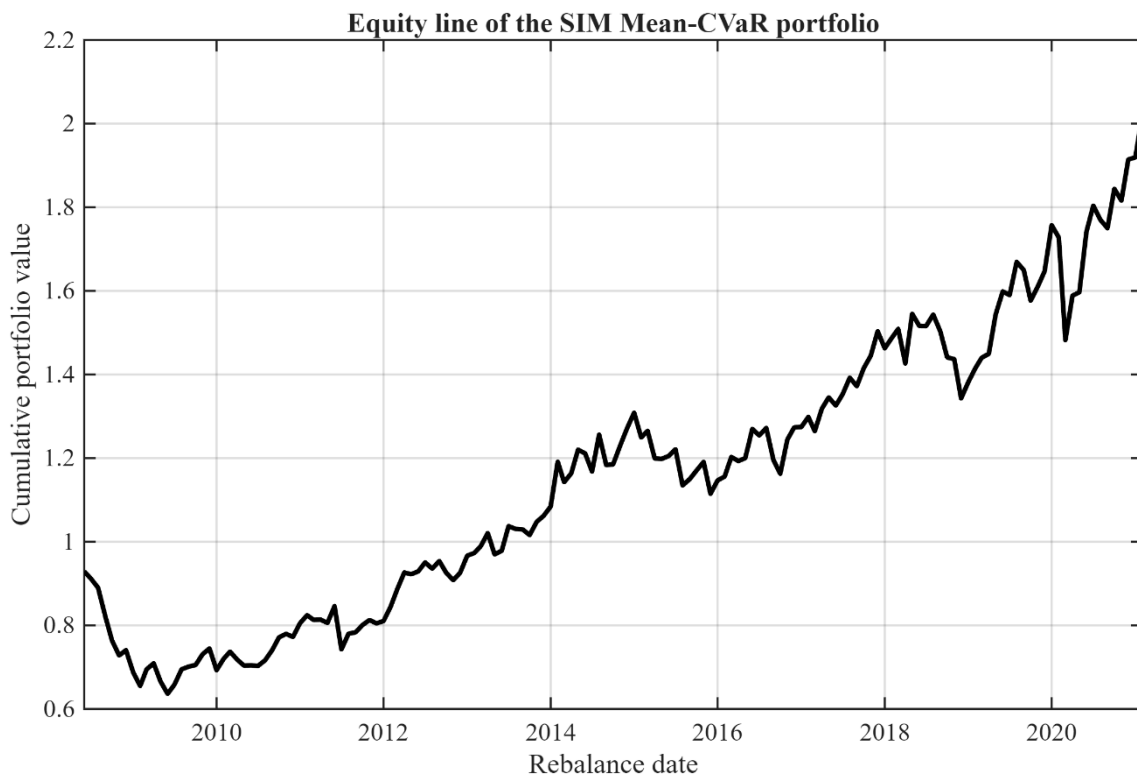
Toward this process it is also possible to reduce the risk of backtest overfitting: portfolio rules that are excessively tailored to historical data may show strong in-sample results but fail to preserve the same risk–return characteristics when applied to out-of-sample data; by applying the procedure across varying market conditions and to previously unseen data, the analysis offers a more stringent test of whether the SIM Mean–CVaR framework can consistently preserve stable risk–return characteristics over time, rather than simply performing well because of patterns specific to the data used for estimation (Bailey & Lopez de Prado, 2021). The subsequent sections present the empirical findings produced by the SIM Mean–CVaR approach across the entire sample period.

### 2.2.1 Portfolio Performance and Downside Risk Dynamics

The long-run performance of the optimized portfolio is assessed by examining the evolution of its equity line, which tracks the cumulative value of an initial

investment of one monetary unit at the outset of the backtest, with subsequent rebalancing based on the portfolio weights obtained from the optimization, then the equity curve offers a time-varying view of portfolio growth, enabling the detection of both sustained periods of appreciation and intervals marked by heightened financial stress (Rockafellar & Uryasev, 2000; Staino & Russo, 2020).

*Figure 2.2.1 - Equity Line of the SIM Mean-CVaR Portfolio*



*Source: author's elaboration using MATLAB outputs.*

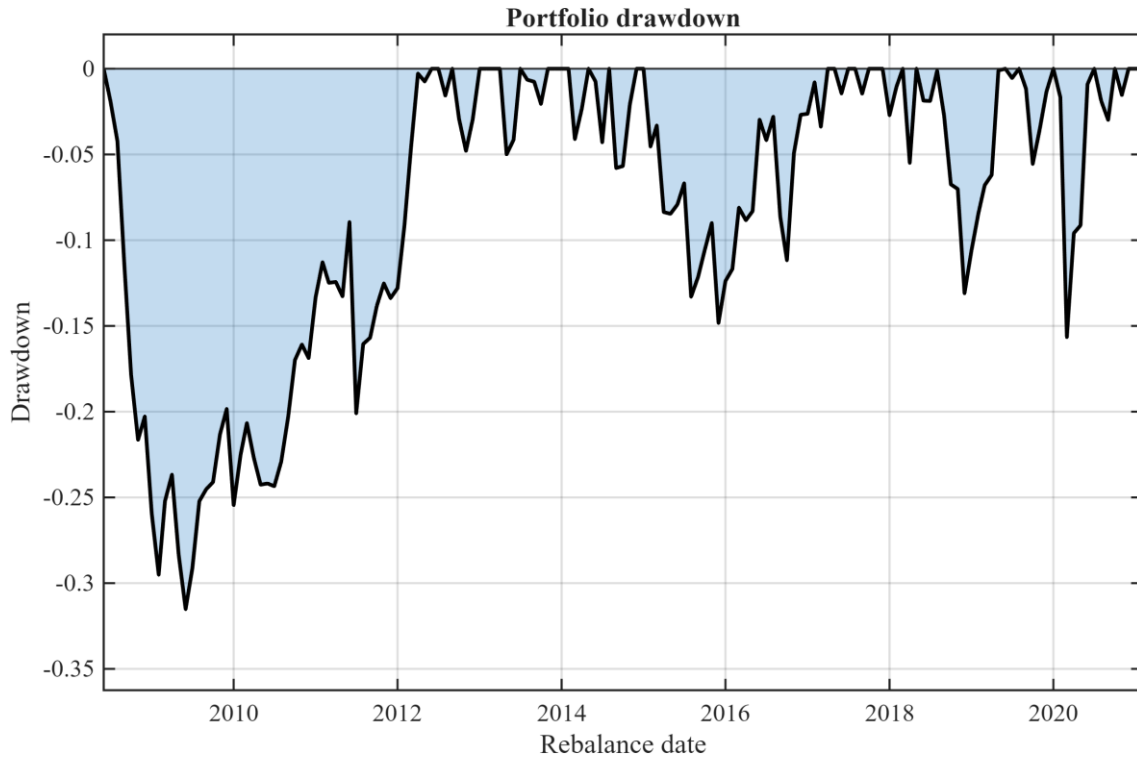
Figure 2.2.1 illustrates the cumulative performance produced by the rolling estimation of the portfolio strategy across the full sample period: in the early part of the sample, the equity curve shows a marked downward trend, consistent with the extreme market instability observed during the 2008–2009 Global Financial Crisis, specifically, in this interval, the cumulative portfolio value drops well below its initial benchmark, underscoring the challenge of capital preservation under conditions of systemic risk, heightened volatility, and a rapid increase in

cross-asset correlations among equities; once the crisis period ends, the portfolio begins to rebound, gradually moving toward a steadier and more sustained upward path (Han et al., 2016).

Starting around 2010, the equity curve displays a relatively steady upward trajectory, with only brief interruptions caused by temporary declines, most notably during the 2015–2016 market slowdown and the volatility triggered by the pandemic in 2020, it's important to notice that despite these interim drawdowns, the cumulative portfolio value continues to rise throughout the observation horizon, culminating in a final level close to 2.0 at the end of the sample period (Cashin et al., 2016; Baker et al., 2020). These outcomes indicate, from an economic point of view that the rolling SIM Mean–CVaR approach sustained positive gains over time, even though it was repeatedly subjected to substantially different market conditions (Staino & Russo, 2020).

Although cumulative performance offers a picture of wealth accumulation, it does not capture the degree and continuity of negative market conditions, as a consequence, the assessment should also track the progression of portfolio drawdowns, defined as the decline in portfolio value from a previously reached peak; drawdown behaviour is inherently path-dependent, making it especially valuable for capital preservation, as it directly measures both the scale and the persistence of the realized losses incurred by the investor (Chekhlov et al., 2005).

Figure 2.2.2 - Drawdown Dynamics of the SIM Mean-CVaR Portfolio



Source: author's elaboration using MATLAB outputs.

Figure 2.2.2 shows the drawdown trajectory associated with the optimized portfolio: looking at the graph it is possible to observe that the largest drawdown occurs in the 2008–2009 financial crisis, when the portfolio declines by more than 30% from its prior peak, a pattern that is consistent with the characteristics of systemic downturns, in which diversification advantages erode because correlations among risky assets rise markedly; consequently, portfolios derived from downside-risk optimization still remain vulnerable to widespread market declines when market stress simultaneously impacts most securities in the considered universe (Chekhlov et al., 2005; Han et al., 2016). Following this stage, drawdowns become substantially less persistent and are typically more limited in magnitude, in this regard, later market disruptions continue to exhibit temporary declines, especially in the 2015–2016 period and during the COVID-19 shock in 2020, although these instances remain markedly smaller than the losses observed during the global financial crisis (Cashin et al., 2016; Baker et al., 2020).

The equity curve and the drawdown profile suggest that the SIM Mean–CVaR strategy faces pronounced stress during systemic crises but can recover over time and sustain a positive long-term growth path under varying market conditions (Bailey et al., 2021; Staino & Russo, 2020).

### 2.2.2 Scenario Distributions and Realized Returns

Following the evidence derived from the equity curve and drawdown behaviour, the analysis below examines the connection between the simulated portfolio return distributions and the realized monthly results recorded in the backtest: in order to realize this comparison three benchmark years (2008, 2013, and 2020) are selected to assess how the SIM-based scenario generation performs across clearly different market conditions, more precisely these years were chosen because they capture three markedly distinct contexts: a system-wide financial crisis, an extended expansionary period, and an exogenous shock followed by swift market repricing (Manda, 2010; Securities and Futures Commission, 2014; Mazur et al., 2021).

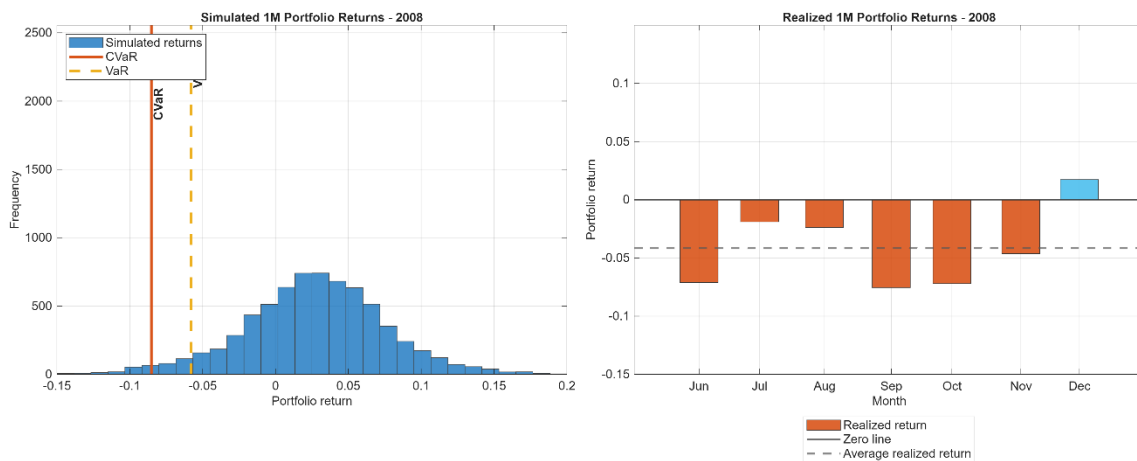
For each year considered, the empirical results are presented using two side by side graphical displays: the left panel reports the simulated distribution of one-month portfolio returns generated through the SIM, a histogram that captures the spread of the portfolio outcomes generated by the optimization procedure, the blue bars indicate the frequency with which the simulated portfolio returns fall within each return interval, while the vertical markers denote the VaR and CVaR values computed at the chosen confidence level; therefore the figure offers an ex-ante representation of the downside-risk characteristics associated with each market scenario. The figure on the right reports the monthly portfolio returns that were actually realized during the corresponding year: each bar shows the realized out-of-sample return for a given month, whereas the horizontal reference lines denote the zero-return benchmark and the average realized monthly return across the year; this layout enables a comparison between the risk conditions generated by the

scenario and the observed trajectory of portfolio performance over the investment horizon (Staino & Russo, 2020; Fairbrother et al., 2022 ).

The comparison between the simulated one-month portfolio return distributions and the actually observed monthly portfolio returns is particularly useful for both risk management and portfolio construction (Rockafellar & Uryasev, 2000): across the years under consideration, the results indicate that the trend of portfolio return differs markedly across market regimes, meaning that periods of crisis are associated with wider, more adversely skewed distributions, while expansionary phases typically produce narrower dispersions and comparatively more stable realized outcomes.

#### 2.2.2.1 2008: Crisis Regime and Downside Pressure

*Figure 2.2.3 - Distribution of Portfolio Returns During the 2008 Financial Crisis*



*Source: author's elaboration using MATLAB outputs.*

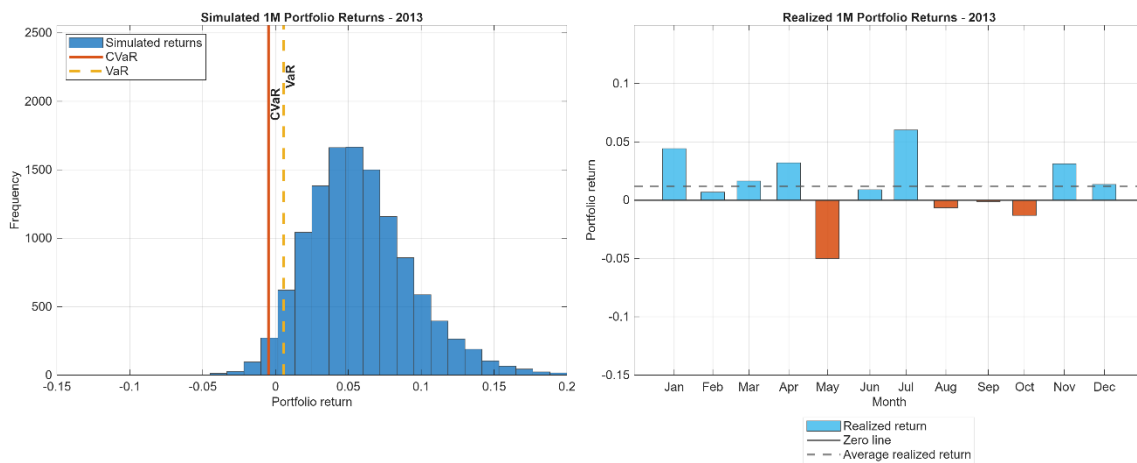
As illustrated in figure 2.2.3, the 2008 outcomes correspond to the worst conditions within the sample period: the simulated return distribution is clearly broader and exhibits a downward shift compared with subsequent years, suggesting that the scenario construction explicitly reflects the marked rise in market uncertainty triggered by the global financial crisis; accordingly both VaR and CVaR fall substantially within the negative domain, thereby confirming the existence of significantly heightened downside risk and a more pronounced exposure to losses

in the left tail.

The realized monthly portfolio returns align with this stressed scenario: in most months, performance is negative, with repeated losses occurring throughout the latter half of the year and only a modest rebound near the end of the period, so this path reflects a sustained worsening of market conditions that persisted across multiple months; this pattern is consistent with the systemic characteristics of the 2008 financial crisis: during periods of pronounced stress, equity correlations typically rise rapidly, the scope for diversification diminishes, and measures of downside risk increasingly reflect wide market movements; as a consequence, the scenario distributions adequately capture a setting marked by heightened tail risk and continuing downward pressure (Rockafellar & Uryasev, 2000; Manda, 2010).

#### 2.2.2.2 2013: Expansionary Conditions and Controlled Risk

*Figure 2.2.4 - Distribution of Portfolio Returns During the 2013 Expansionary Market Phase*



*Source: author's elaboration using MATLAB outputs.*

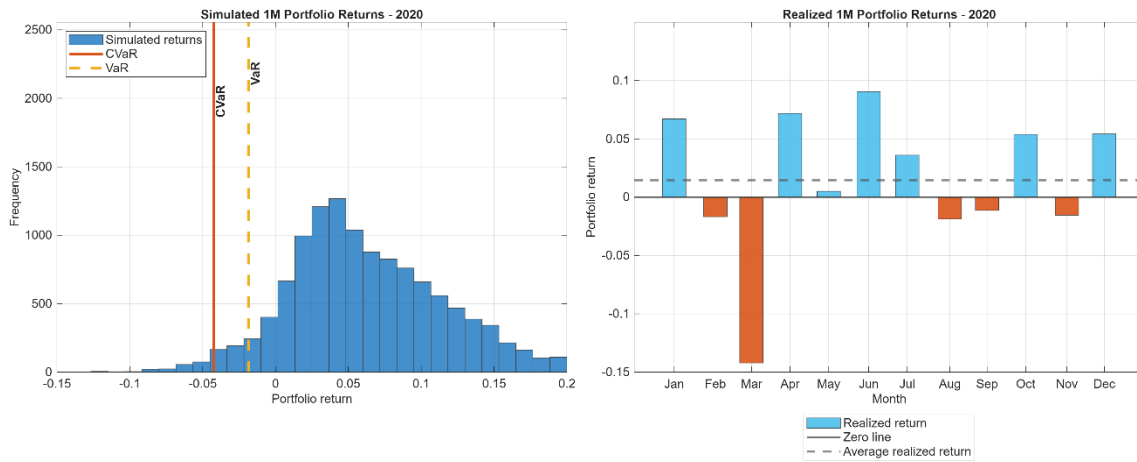
Compared with 2008, as shown in figure 2.2.4, the 2013 distributions exhibit a markedly more stable pattern: the simulated returns cluster within a narrower interval and shift toward positive outcomes, whereas both VaR and CVaR remain close to zero compared with crisis conditions, suggesting that the optimization procedure selected a market regime featuring more limited downside exposure and reduced overall uncertainty. The observed portfolio returns lend support to this conclusion, showing that in the majority of months, the realized results are

positive, while negative months are relatively infrequent and show limited magnitude; accordingly, the mean realized performance remains consistently above zero over the course of the year, while the dispersion of realized returns is markedly lower than that recorded in 2008, suggesting a more uniform and stable pattern of performance (Rockafellar & Uryasev, 2000; Staino & Russo, 2020).

This pattern is consistent with the wider macro-financial conditions observed in 2013, marked by accommodative stance of monetary policy, relatively limited volatility, and the continued strength of U.S. equity markets; then under such circumstances the likelihood of extreme downside tail events is reduced and return behaviour across securities remain comparatively stable (Securities and Futures Commission, 2014).

### 2.2.2.3 2020: Pandemic Shock, Asymmetry, and Rapid Repricing

*Figure 2.2.5 - Distribution of Portfolio Returns During the COVID-19 Market Crash*



*Source: author's elaboration using MATLAB outputs.*

Looking at figure 2.2.5, the 2020 results show a pronouncedly imbalanced profile compared with 2008 and 2013, because while the simulated distribution remains located in positive values, it also features a marked left tail and a considerably larger spread than in years characterized by greater stability, suggesting that, during the COVID-19 period, the scenario generation process captured the joint presence of recovery prospects and substantial downside risk (Fairbrother et al., 2022; Mazur et al., 2021). This dual trajectory is evident in the realized portfolio

returns in the first month, they decline sharply following a negative shock associated with the pandemic market collapse, while the subsequent months show marked and positive recoveries, consequently, as opposite to the steady, protracted deterioration recorded in 2008, the 2020 episode produces a sharp discontinuity in returns, followed by a gradual but more rapid return to stability (Mazur et al., 2021; Staino & Russo, 2020). This dynamic aligns with the economic features of the COVID-19 shock: it triggered one of the sharpest equity market declines in recent financial history, yet it was rapidly followed by unusual monetary and fiscal measures, as a consequence, trading conditions shifted quickly from widespread market losses to strong recovery expectations, generating pronounced asymmetry in both simulated and observed return distributions (Mazur et al., 2021; Rockafellar & Uryasev, 2000).

### 2.2.3 Sector Allocation Dynamics

A further element to consider when analysing the behaviour of the strategy is the temporal change in portfolio allocations across GICS sectors, sector-level weights provide additional information on how the optimizer reallocates capital among different economic segments during the rolling backtest, specifically sector allocations may be understood as a higher-level reflection of the portfolio construction mechanism under evolving market conditions (Rockafellar & Uryasev, 2000; Staino & Russo, 2020).

*Table 2.2.1 - Sector Allocation Statistics of the SIM Mean–CVaR Portfolio*

| GICS Sector                   | Average Weight | Maximum Weight |
|-------------------------------|----------------|----------------|
| <b>CONSUMER DISCRETIONARY</b> | 16,54%         | 72,56%         |
| <b>INDUSTRIALS</b>            | 15,11%         | 57,73%         |
| <b>HEALTHCARE</b>             | 14,15%         | 52,88%         |
| <b>INFORMATION TECHNOLOGY</b> | 12,89%         | 49,75%         |
| <b>REAL ESTATE</b>            | 9,41%          | 39,08%         |
| <b>FINANCIALS</b>             | 9,32%          | 39,65%         |
| <b>UTILITIES</b>              | 7,31%          | 34,11%         |
| <b>CONSUMER STAPLES</b>       | 7,25%          | 39,55%         |
| <b>MATERIALS</b>              | 4,39%          | 35,99%         |
| <b>ENERGY</b>                 | 2,38%          | 27,66%         |
| <b>COMMUNICATION SERVICES</b> | 1,26%          | 8,88%          |

*Source: author's elaboration using MATLAB outputs.*

Table 2.2.1 reports the average and maximum portfolio weights assigned to each GICS sector across the full sample. The evidence indicates that the portfolio does not follow a fixed sector allocation; rather, investments are periodically rebalanced in response to the evolving trade-off between expected returns and downside risk, as implied by the optimization process, then in this setting, several sectors frequently account for substantial allocation weights, although they do not always maintain the same weight and importance in the portfolio. (Bessler et al., 2021; Alexiou et al., 2020). Consumer Discretionary shows the highest mean allocation (16.54%) and the highest peak (72.56%), showing that, in particular market conditions, the optimization approach consistently highlighted especially advantageous investment opportunities within industries linked to cyclical consumer spending; Industrials is also prominent across the full sample, exhibiting an average allocation above 15% and a peak of more than 57%, which indicates that the strategy often benefited from sectors associated with economic recovery and expectations of expansion. Moreover, Healthcare is another significant element of the portfolio, with an average allocation exceeding 14%, in particular way, compared with more cyclical sectors, Healthcare generally shows relatively steadier earnings patterns and defensive features, which can strengthen the portfolio's risk-return profile when uncertainty rises; in parallel, Information Technology constitutes a notable component of the allocation, with maximum exposures nearing 50%, indicating periods during which the market favoured growth and innovation oriented factors (Bessler et al., 2021; Alexiou et al., 2020). At the same time, Financials and Real Estate exhibit intermediate average weights close to 9%, whereas Utilities, Consumer Staples, Materials, and Energy make more selective allocations depending on the prevailing market regime; by contrast, Communication Services remains negligible across most of the sample, both in average terms and at its maximum exposure, then the allocation procedure does not appear to be excessively focused on a single persistent macroeconomic driver (Bessler et al., 2021; Alexiou et al., 2020). A notable finding is that diversification

occurred alongside brief periods of increased investment in specific sectors: some sectors reach high weights at certain times, but they do not remain dominant throughout the sample period, instead, leadership shifts between sectors over time, indicating that the optimization procedure adjusts exposures in response to evolving market condition; the observed concentration can be seen as a tactical response that emerges only when the expected reward relative to downside risk becomes sufficiently favourable for a specific sector (Rockafellar & Uryasev, 2000; Alexiou et al., 2020). On the whole, the optimization process reallocates exposure toward sectors with more favourable simulated distributions, while reducing allocations to segments characterized by weaker expected performance. (Rockafellar & Uryasev, 2000; Ferreira et al., 2021).

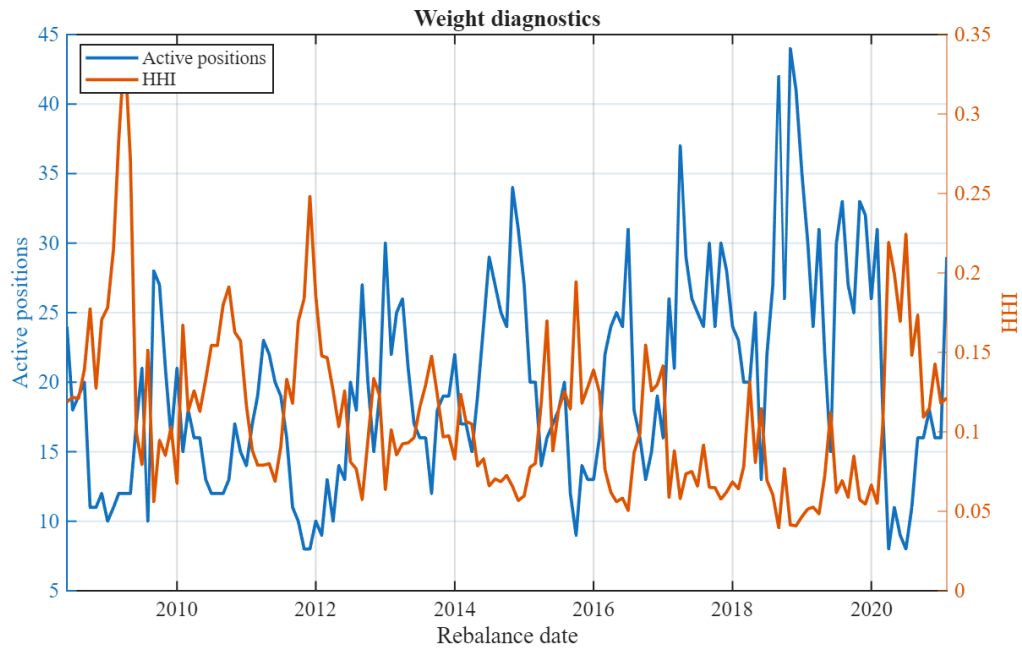
#### 2.2.4 Weight Concentration and Portfolio Breadth

A further insight into the configuration of the implemented strategy is obtained by examining how portfolio breadth and weight concentration change over time, focusing on two complementary measures evaluated at each rebalancing date: the number of active holdings, defined as the securities with a strictly positive portfolio weight, and the Herfindahl–Hirschman Index (HHI), computed from the portfolio-weight vector as:

$$HHI_t = \sum_{i=1}^N w_{i,t}^2$$

where  $w_{i,t}$  denotes the portfolio weight assigned to security  $i$  at rebalancing date  $t$  (Anand & Ramasubramanian, 2015; Fulkerson & Riley, 2019).

Figure 2.2.6 - Weight Concentration and Portfolio Breadth



Source: author's elaboration using MATLAB outputs.

In figure 2.2.6, it is possible to observe the joint dynamics of the two indicators over the rolling backtesting period, illustrating that the number of active positions changes across rebalancing dates: it alternates between relatively concentrated holdings and more diversified allocations that include a substantially larger number of securities.

The HHI series supports this interpretation by capturing how capital is allocated across the selected assets: lower HHI values reflect more balanced distributions, whereas higher values signal periods in which portfolio weights concentrate within a narrower set of holdings (Anand & Ramasubramanian, 2015), in addition the index typically stays at moderate levels throughout most of the sample, despite the appearance of temporary concentration phases at particular points in time, suggesting that the optimization strategy does not consistently rely on highly concentrated allocations; the figure highlights an inverse co-movement between the two indicators, meaning that intervals with a greater number of active positions are typically associated with lower concentration levels, whereas periods characterized by fewer holdings tend to correspond to higher HHI values.

Economically this relationship is plausible: as the optimizer selects a narrower set of relatively attractive investment opportunities, capital concentration naturally increases (Anand & Ramasubramanian, 2015; Fulkerson & Riley, 2019).

#### 2.2.5 Discussion of results

The rolling implementation of the SIM Mean–CVaR methodology enables the portfolio strategy to be assessed over an extended horizon, so that its statistical properties and allocation patterns can be tracked as they change over time; the analysis presented in this chapter focuses on the way realized results, downside-risk exposure, portfolio concentration, and sector composition jointly evolve throughout the entire backtesting window (Rockafellar & Uryasev, 2000; Staino & Russo, 2020).

*Table 2.2.2 - Summary Statistics of the SIM Mean–CVaR Rolling Backtest*

|                                         |         |
|-----------------------------------------|---------|
| <b>Months_Backtested</b>                | 153     |
| <b>Annualized return</b>                | 5,69%   |
| <b>Annualized volatility</b>            | 13,81%  |
| <b>Sharpe ratio</b>                     | 0,41    |
| <b>Average monthly return</b>           | 0,54%   |
| <b>Monthly volatility</b>               | 3,99%   |
| <b>Maximum drawdown</b>                 | -31,52% |
| <b>Best monthly return</b>              | 9,88%   |
| <b>Worst monthly return</b>             | -14,22% |
| <b>Positive months ratio</b>            | 61,44%  |
| <b>Average scenario expected return</b> | 4,76%   |
| <b>Average VaR Loss (95%)</b>           | 0,40%   |
| <b>Average CVaR Loss (95%)</b>          | 1,43%   |
| <b>Average active positions</b>         | 19,88   |
| <b>Average Max weight</b>               | 19,93%  |
| <b>Average HHI</b>                      | 0,11    |

*Source: author's elaboration using MATLAB outputs.*

Table 2.2.2 presents the main figures related to the rolling implementation of the strategy and summarizes the portfolio's performance, risk, and composition over the 153 months analysed in this study (Staino & Russo, 2020). The portfolio achieved an annualized return of 5.69% and an average monthly return of 0.54%, although it experienced some short periods of volatility, it maintained positive

long-term growth throughout the observation period; moreover, the fact that more than 61% of months were positive indicates that the portfolio's gains occurred regularly over time rather than being limited to a few isolated periods. At the same time, the results indicate that portfolio growth entails a substantial exposure to downside risk: the maximum drawdown is greater than 31%, and the most adverse month shows a realized loss exceeding 14%, evidencing the strategy's vulnerability to episodes of systemic market stress, that remain economically reasonable, since the sample comprises several of the most turbulent market phases in recent decades; the mean VaR and CVaR losses remain below the most severe observed losses, suggesting that extreme losses mainly occur during exceptional market crises rather than under normal market conditions (Rockafellar & Uryasev, 2000; Luan et al., 2022).

The statistics describing the portfolio composition also yield informative evidence on the dynamics of the optimization procedure: on average, the strategy spreads its capital across approximately 20 active holdings, indicating a portfolio that remains selective without becoming overly concentrated, a finding confirmed by the mean HHI of 0.11, which reflects a moderate degree of weight concentration across most observations; parallel to it, the average maximum individual position, which is close to 20%, indicates that the optimizer sometimes assigns greater weight to certain assets, while still maintaining a diversified portfolio (Anand & Ramasubramanian, 2015; Fulkerson & Riley, 2019).

As outlined in the preceding sections, an additional key aspect concerns the link between simulated and realized performance measures, in particular based on the scenarios generated by the model, on average, a positive result is expected during the backtest; however, realized returns necessarily deviate from simulated expectations during certain periods, despite these differences the empirical results indicate that the simulations captures economically meaningful information regarding the prospective distribution of portfolio risk and return (Staino & Russo, 2020; Fairbrother et al., 2022).

Taken together, the results demonstrate that the SIM Mean–CVaR strategy

produces portfolio outcomes that vary considerably over time, highlighting the complexity and volatility of the market dynamics.

### **3 Theoretical Foundation of the Multi-Index Model**

#### **3.1 Introduction and Motivation for the Multi-Index Model**

The Single-Index Model provides a simple description of how security returns move together by assuming that their relationship is mainly influenced by a single common market factor, an approach that, as noted in the previous chapter, facilitates the estimation of the covariance matrix and enables a clear separation of total risk into systematic and idiosyncratic components; therefore, because each security is related only to the market portfolio through its beta, the model explains a large proportion of the variation in returns without requiring extensive estimation (Elton et al., 2014). Although such a single systematic-factor assumption offers practical advantages, it may become overly restrictive when the investment universe is wide and the firms it contains are heterogeneous: in real financial markets, companies are not driven solely by broad market conditions, but they are often subject to additional shared influences tied to the industries and sectors in which they operate; accordingly firms within the same economic segment tend to face comparable production setups, demand patterns, regulatory regimes, technological developments, and differential exposure to macroeconomic shocks. Therefore, securities from related sectors may continue to move together even after general market effects are considered (Chan et al., 2007; Fama & French, 2015). The empirical results reported in the previous chapter support this interpretation: the residual correlation analysis showed that firms in several GICS sectors remain connected, indicating that their residuals are not fully independent.

This highlights that the one-factor model cannot fully capture the complex covariance structure observed in the data and may be limited in representing how risk is transmitted across firms and sectors; from a portfolio management standpoint, this constraint is particularly important because the accuracy of covariance estimation determines the quality of risk quantification, the diversification characteristics of the portfolio, and the robustness of allocation choices (Elton et al., 2014; Fama & French, 2015).

To overcome these limitations, the analysis is expanded to a Multi-Index Model in which stock returns are permitted to co-vary not only with the overall market portfolio but also with additional systematic components linked to sectoral developments: the Multi-Index specification separates the main sources of co-movement into several risk channels; for instance, these elements may reflect contemporaneous interrelationships within industries, different sensitivities to macroeconomic conditions across sectors, and the transmission of shocks through sectoral balance-sheet and demand mechanisms (Chan et al., 2007; Elton et al., 2014). In other words, in this context, risk transmission is not attributed to a single uniform source, but it emerges from multiple economic factors that influence different groups of firms according to their sectoral exposure and organisational characteristics, this approach is especially important during phases of market stress, when shocks tend to spread asymmetrically across industries, producing correlation structures that a one-factor approach is unable to capture adequately (Fama & French, 2015).

### 3.2 Methodological Setting of the Multi-Index Model

Operationally, the Multi-Index approach used in this study separates market-wide risk from sector-specific risk, it does this through a two-step regression process that first identifies the effects of the overall market and then examines sectoral influences (Elton et al., 2014; Fama & French, 2015).

Under the model's general specification, the return on security  $i$  can be expressed as:

$$R_i = a_i + b_{i1}I_1 + b_{i2}I_2 + \dots + b_{iL}I_L + c_i$$

in this formulation  $a_i$  identifies the intercept term,  $I_1, \dots, I_L$  correspond to the systematic indexes included in the model, while  $b_{ij}$  captures the degree to which security  $i$  is sensitive to index  $j$ , and  $c_i$  represents the residual component reflecting firm-specific influences (Elton et al., 2014).

In this specific setting, the model is applied using a two-index structure: the first index captures the aggregate market portfolio, measured by the S&P 500, whereas

the second index captures the sector factor corresponding to the GICS sector of the firm; consequently, the aim is to extract the component of sector returns that persists after controlling for, and thereby removing, the overall market influence (Elton et al., 2014); accordingly, the return equation may be expressed as:

$$R_i = a_i + b_{iM}I_M + b_{iS}I_S + c_i$$

where  $I_M$  denotes the market index, and  $I_S$  refers to the orthogonalized sector index linked to the sector classification of security  $i$ , the coefficient  $b_{iM}$  quantifies the stock's responsiveness to overall market movements, while  $b_{iS}$  measures the firm's exposure to sector-specific systematic dynamics after excluding the common market component; lastly, the residual term  $c_i$  captures the idiosyncratic portion of returns that is not accounted for by either of the systematic factors (Elton et al., 2014; Herskovic et al., 2016).

For the purposes of this analysis the sector index is defined as the equally weighted average return of all stocks within the same GICS sector as security  $i$ , in addition to prevent the dependent variable from being mechanically incorporated into its own explanatory component, the return of security  $i$  is omitted from the sector aggregation whenever other firms in the same sector are available; accordingly, the constructed factor is intended to capture the common sectoral dynamics characterising firms within the same economic segment (Chan et al., 2007; Alexiou et al., 2020).

The first stage of the estimation procedure consists in orthogonalizing the sector factor with respect to the aggregate market index; specifically correlated indexes are transformed into orthogonal factors through a regression-based decomposition procedure in which the component explained by the market factor is removed from the original sector index (Elton et al., 2014; Gribisch et al., 2020). The orthogonalization step can be represented as:

$$I_S = d_t = I_S^* - \gamma_0 - \gamma_1 I_M$$

where  $I_S^*$  denotes the original sector index,  $I_M$  is the aggregate market index, and  $\gamma_0$  and  $\gamma_1$  represent the regression coefficients obtained in the orthogonalization step, while the variable  $d_t$  corresponds to the regression residual and thus captures

the portion of sector returns that is not attributable to broad market movements; as a consequence, the adjusted sector index is uncorrelated with the market portfolio by construction (Elton et al., 2014; Gribisch et al., 2020).

After obtaining the orthogonalized sector factors, the second step involves estimating the return equation for each individual security: the return of every stock is modelled by regressing it on the market index and only on the orthogonalized sector factor corresponding to its own GICS sector; then the final result implies that each security is characterized by exposure to just two systematic risk components: the overall market factor and the specific factor related to the sector in which the firm operates (Elton et al., 2014; Fama & French, 2015).

Given the orthogonality conditions assumed in the first stage of the procedure, the variance of security returns can be expressed as the sum of the contributions from the market factor, the sector factor, and the idiosyncratic residual component; accordingly, the variance of security  $i$  is given by:

$$\sigma_i^2 = b_{iM}^2 \sigma_M^2 + b_{iS}^2 \sigma_S^2 + \sigma_{c_i}^2$$

where the first term denotes the contribution of aggregate market risk, the second term measures the share attributable to systematic sector fluctuations, and the last term corresponds to the remaining idiosyncratic variance not captured by the model (Elton et al., 2014).

In this context, the covariance between two assets reflects the degree to which they are jointly exposed to the same systematic factors, then under the two-index structure here employed, the covariance can therefore be expressed as:

$$\sigma_{ij} = b_{iM} b_{jM} \sigma_M^2 + b_{iS} b_{jS} \sigma_S^2$$

Consequently, firms within the same industry may still display pronounced co-movement even after accounting for general market influences, as they continue to share exposure to the same orthogonalized sector factor, therefore this decomposition establishes the methodological basis for the empirical assessment of the correlation structures implied by the Single Index and Multi-Index models introduced in the subsequent sections (Chan et al., 2007; Elton et al., 2014).

Table 3.2.1 - Illustrative Estimation Results of the Multi Index Model

| Stock                                        | GICS Sector            | Alpha   | Beta Market | Beta Own Sector | Sector Variance Component | Variance | R2     | Delta R2 vs SIM |
|----------------------------------------------|------------------------|---------|-------------|-----------------|---------------------------|----------|--------|-----------------|
| Camden Property Trust (NYSE:CPT)             | REAL ESTATE            | 0,0001  | 0,4784      | 0,2961          | 0,000001821               | 0,0002   | 0,2013 | 0,0087          |
| Ventas, Inc. (NYSE:VTR)                      | REAL ESTATE            | 0,0006  | 0,8364      | 0,3697          | 0,000002791               | 0,0004   | 0,2967 | 0,0065          |
| Apple Inc. (NasdaqGS:AAPL)                   | INFORMATION TECHNOLOGY | 0,0004  | 1,1634      | -0,4603         | 0,000002526               | 0,0005   | 0,5150 | 0,0054          |
| KLA Corporation (NasdaqGS:KLAC)              | INFORMATION TECHNOLOGY | 0,0005  | 0,8413      | 0,0786          | 0,000000072               | 0,0003   | 0,4120 | 0,0002          |
| The Boeing Company (NYSE:BA)                 | INDUSTRIALS            | 0,0000  | 0,9966      | 0,8834          | 0,000010522               | 0,0004   | 0,5297 | 0,0300          |
| The TJX Companies, Inc. (NYSE:TJX)           | CONSUMER DISCRETIONARY | 0,0001  | 0,7195      | -0,0027         | 0,000000000               | 0,0004   | 0,2478 | 0,0000          |
| Arthur J. Gallagher & Co. (NYSE:AIG)         | FINANCIALS             | 0,0002  | 0,8356      | 0,7101          | 0,000006389               | 0,0003   | 0,4851 | 0,0239          |
| Cadence Design Systems, Inc. (NasdaqGS:CDNS) | INFORMATION TECHNOLOGY | -0,0002 | 1,2985      | 1,5811          | 0,000027158               | 0,0007   | 0,4384 | 0,0366          |
| Autodesk, Inc. (NasdaqGS:ADSK)               | INFORMATION TECHNOLOGY | -0,0001 | 0,5245      | 0,1306          | 0,000000195               | 0,0007   | 0,0751 | 0,0003          |
| APA Corporation (NasdaqGS:APA)               | ENERGY                 | -0,0007 | 1,7116      | 1,2119          | 0,000030251               | 0,0016   | 0,3515 | 0,0194          |

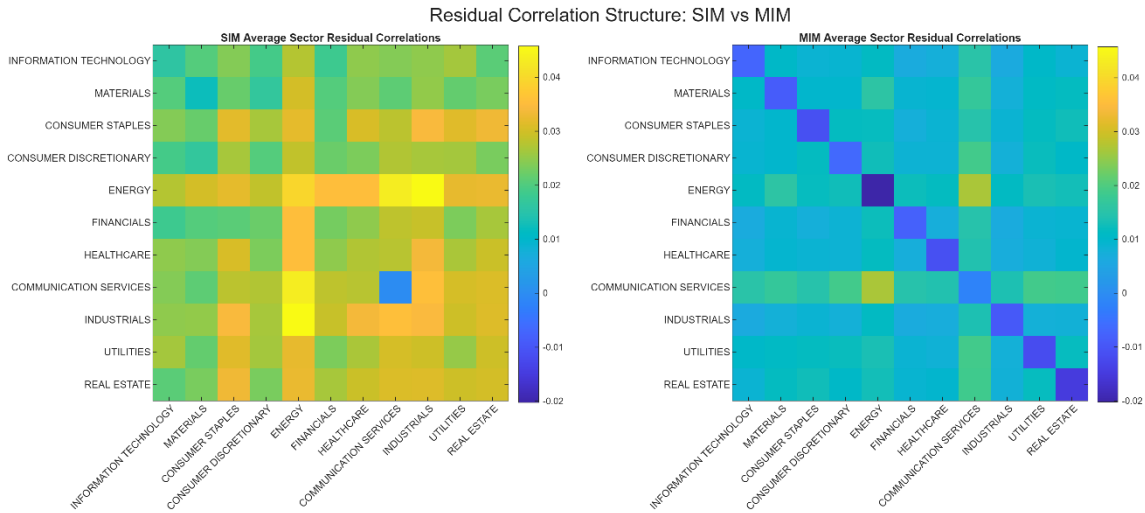
Source: author's elaboration using MATLAB outputs

Table 3.2.1 presents a selection of estimated parameters from the Multi-Index Model for a group of securities from multiple GICS sectors, specifically, it reports the estimated market betas, sector betas, variance components, and the explanatory power obtained under the two-index specification considered (Elton et al., 2014). The findings show that, in addition to overall market exposure, several securities are strongly affected by the orthogonalized sector factor, this confirms that sectoral factors influence securities alongside the common market factor (Chan et al., 2007; Fama & French, 2015); however, the strength of these factor sensitivities varies substantially across firms, suggesting differences in the degree to which sector-level effects contribute to the formation of returns. Moreover, the positive  $\Delta R^2$  values reported for most securities indicate that incorporating the orthogonalized sector component typically enhances the model's explanatory capacity when compared with the Single Index specification (Elton et al., 2014).

### 3.3 Comparison of sector correlation structure: SIM vs MIM

To further examine the implications of the Multi-Index Model for the dependence structure of equity returns, figure 3.3.1 contrasts the average residual sector correlation matrices derived under the Single Index and Multi-Index frameworks: the analysis is limited to residual correlations, that is, the component of joint return variation that persists after accounting for the systematic elements captured by each model (Elton et al., 2014; Fama & French, 2015).

Figure 3.3.1 - Residual Sector Correlation Structure: SIM vs MIM



Source: author's elaboration using MATLAB outputs

More specifically, each entry in the matrices denotes the mean residual correlation estimated across pairs of securities within the corresponding GICS sectors, the graphical comparison offers an aggregate view of the extent to which each methodological approach succeeds in separating the shared systematic components from the remaining idiosyncratic portion of returns.

Under the Single Index specification, residual correlations remain positive across several sector combinations, even after controlling for the market factor, which means that the market explains a part of the co-movement among securities, but does not account for all of it; in this regard, the off-diagonal elements of the matrix continue to indicate moderate dependence among economically distinct sectors (Elton et al., 2014; Wang et al., 2020).

Examining the SIM residual matrix reveals that the strongest residual interactions are primarily concentrated in the Energy, Industrials, Utilities, and Consumer Staples sectors; specifically, the Energy sector is associated with comparatively higher correlation residuals with Industrials, Utilities, and Financials, whereas Industrials continue to exhibit consistently positive interactions with multiple cyclical sectors; moreover, Consumer Staples and Real Estate show residual dependence across many sector pairs, indicating that the remaining co-movement is widespread rather than limited to a few sector clusters; at the same time, the SIM

residual matrix shows a relatively stable correlation pattern across several sectors, suggesting that the residuals still include some common market movements and are not entirely idiosyncratic. Therefore, the SIM shows that residual interdependence is widespread across the correlation matrix (Fama & French, 2015; Elton et al., 2014; Wang et al., 2020).

On the other hand, the residual correlation structure estimated within the Multi-Index Model is different from that obtained in the SIM specification: it is characterized by lower overall correlation level across most sector pairs, and the resulting matrix appears more compact and uniform; the attenuation of residual dependence is particularly clear in several off-diagonal blocks: once the orthogonalized sector factors are incorporated into the estimation, the remaining correlations within these blocks decline substantially (Elton et al., 2014; Fama & French, 2015).

More specifically, the residual interactions previously identified between Energy and several cyclical sectors become weaker under the MIM, whereas the broad dependence patterns linked to Industrials, Utilities, and Financials are considerably reduced, at the same time a similar result emerges for Real Estate and Consumer Staples, for which the residual correlation structures are more concentrated and show less persistence across the matrix; several diagonal blocks move closer to zero residual dependence, which implies that the orthogonalized sector component in the MIM directly captures part of the common variation within sectors that are incorporated in the SIM residuals (Elton et al., 2014); distinct structural patterns emerge across the remaining correlation results: within the SIM, some sector groups still show relatively stable residual interactions, especially among industries linked through integrated economic activities or partially overlapping risk profiles; by contrast, under the MIM, the residual correlation distribution appears more homogeneous and exhibits weaker dominance by clearly defined sector blocks, suggesting a more robust distinction of shared systematic influences from idiosyncratic fluctuations. Essentially, the MIM reshapes the structure of residual dependence, in particular, the resulting correlation matrix shows lower

persistence, less clustering, and a more uniform distribution of residual correlations across sectors, confirming that this model provides a more detailed view of return dependence among securities (Elton et al., 2014; Fama & French, 2015; Wang et al., 2020).

### 3.4 Sectoral Dependence and Portfolio Diversification

The previous findings have important implications for portfolio construction: diversification benefits depend not only on the number of assets in a portfolio, but also on how the relationships between their returns are modelled (James et al., 2022); therefore a portfolio may look well diversified on the basis of a large number of holdings, but are still subject to the same underlying risk factors if those assets are driven by similar macroeconomic conditions, implying that diversification should be understood not merely as a quantitative notion, but as a reflection of how risk is distributed across sources of variation that are independent or only partially independent (Liu et al., 2018; James et al., 2022). Sectoral dependence is important in this context because firms in different industries may respond differently to general economic shocks, however, companies operating in the same or closely related sectors generally face similar conditions throughout the business cycle, meaning that they are jointly exposed to regulatory developments, movements in aggregate demand, comparable patterns in input costs, and shared financing constraints (Liu et al., 2018; Tulloch et al., 2020); combining securities may therefore overstate the diversification gain when residual correlation is not explicitly addressed (Wang et al., 2020; James et al., 2022).

Comparing the residual correlation structures of the SIM and MIM provides further evidence that the specification of correlation among security returns affects the assessment of portfolio risk: in the one-factor model, some common dependence remains in the residuals, making certain exposures appear more idiosyncratic than they are, the MIM instead attributes part of this dependence to specific systematic factors, offering a clearer view of the main sources of portfolio risk (Elton et al.,

2014; Fama & French, 2015). Such distinction is important for covariance estimation, because the covariance matrix constitutes the core input used to quantify portfolio risk, then inaccuracies in the assumed dependence structure can propagate into subsequent allocation decisions; more precisely, if dependence associated with sector factors are understated, an optimization procedure may allocate disproportionately large weights to securities that appear weakly linked in the model, although they face the same sectoral shocks, while a more comprehensive risk specification can clarify cases in which securities that seem unrelated actually share a common source of volatility (Bessler et al., 2021).

The MIM provides a clearer view of how risk is distributed across the portfolio, allowing portfolio risk to be assessed more accurately, especially when sectors respond differently to macroeconomic and financial shocks (Fama & French, 2015; Tulloch et al., 2020); this approach is especially useful during periods of market turbulence: when financial stress increases, correlations often become stronger, shocks can spread unevenly across industries and some sectors are more affected than others because of their exposure to credit conditions, commodity prices, interest rates, consumer demand, and technological developments (Liu et al., 2018; Tulloch et al., 2020).

We can draw the conclusion that including sector-specific factors improves the link between risk modelling and portfolio allocation (Fama & French, 2015; Bessler et al., 2021): by providing a clearer structure for residual risk and a more detailed view of the covariance matrix, the MIM provides a better basis for the subsequent optimization process; this feature is particularly significant in the following chapter, where the Mean-CVaR portfolio optimization approach is implemented using the Multi-Index specification, enabling a direct assessment of the outcomes relative to those obtained earlier under the Single Index Model.

## **4 Portfolio Optimization with Mean-CVaR based on the Multi-Index Model**

Building upon the theoretical foundation established in the preceding chapter, this section implements the Multi-Index Model within the Mean-CVaR portfolio optimization context, the analysis focuses on assessing how the introduction of sectoral systematic components influences the formation and subsequent evaluation of the optimized portfolios; more specifically, the Multi-Index Model serves as the basis for scenario generation in the Mean-CVaR methodology, on the one hand the optimization setup is unchanged relative to the version previously implemented with the SIM, on the other hand the return scenarios are now derived from a more informative factor structure, precisely, each stock return is modelled as depending not only on the overall market factor but also on an orthogonalized sector component specific to each firm, allowing to examine whether the additional information contained in the MIM leads to different portfolio weights, different estimates of downside risk, and, consequently, different out-of-sample performance (Rockafellar & Uryasev, 2000; Elton et al., 2014).

The present chapter begins by illustrating how the Multi-Index structure shapes simulated return distributions using an individual stock example, the discussion then proceeds to the design of the MIM-based Mean-CVaR portfolio, focusing on the scenario distribution, the resulting optimal weights, and the sector allocation; lastly, the rolling backtest outcomes are set alongside those produced under the SIM setting, enabling a direct assessment of whether the more comprehensive modelling of systematic risk translates into improved realized performance and a more favourable risk profile (Rockafellar & Uryasev, 2000; Staino & Russo, 2020; Fairbrother et al., 2022).

### **4.1 Scenario Generation under the Multi-Index Framework**

This analysis examines the empirical features of the return scenarios generated under the MIM specification, the purpose of this section is to assess how the introduction of orthogonalized sector components alters the statistical and

economic characteristics of the simulated return distributions when compared with the SIM (Elton et al., 2014); to deal with this point, the assessment first reviews the estimation outcomes under the Multi-Index specification, placing particular emphasis on those securities for which explanatory power improves the most; accordingly, the return distributions simulated under the SIM and MIM are evaluated at the level of individual stocks to determine the extent to which the more elaborate factor structure reshapes the distributions' profile, dispersion, and downside dynamics, the section then concludes by considering the overall distribution of simulated portfolio returns obtained from the MIM Mean-CVaR specification (Staino & Russo, 2020; Fairbrother et al., 2022).

#### 4.1.1 Estimation Results under the Multi-Index Model

Once the Multi-Index Model has been estimated over the rolling window, its explanatory performance can be compared with that of the Single-Index Model, in particular, the analysis focuses on the securities that benefit most from the inclusion of sectoral systematic components, as indicated by the increase in the coefficient of determination between the SIM and MIM (Elton et al., 2014; Fama & French, 2015).

*Table 4.1.1 - Top 15 Stocks Ranked by Improvement in Explanatory Power under the Multi-Index Model*

| Stock                                                     | GICS Sector      | Alpha   | Beta Market | Beta Own Sector | R <sup>2</sup> | DeltaR <sup>2</sup> vs SIM | Market Risk Share | Sector Risk Share | Residual Risk Share |
|-----------------------------------------------------------|------------------|---------|-------------|-----------------|----------------|----------------------------|-------------------|-------------------|---------------------|
| Charles River Laboratories International, Inc. (NYSE:CRL) | HEALTHCARE       | -0.0003 | 1.1449      | 2.2426          | 0.5578         | 0.1215                     | 43.63%            | 12.15%            | 44.22%              |
| Johnson & Johnson (NYSE:JNJ)                              | HEALTHCARE       | -0.0001 | 1.0325      | 2.0283          | 0.4859         | 0.1070                     | 37.89%            | 10.70%            | 51.41%              |
| Cencora, Inc. (NYSE:COR)                                  | HEALTHCARE       | -0.0001 | 1.0861      | 1.9118          | 0.5077         | 0.0943                     | 41.34%            | 9.43%             | 49.23%              |
| Biogen Inc. (NasdaqGS:BIIB)                               | HEALTHCARE       | 0.0000  | 1.0650      | 1.6501          | 0.5210         | 0.0791                     | 44.19%            | 7.91%             | 47.90%              |
| IDEX Corporation (NYSE:IEX)                               | INDUSTRIALS      | -0.0001 | 1.1718      | 1.7383          | 0.5129         | 0.0717                     | 44.11%            | 7.17%             | 48.71%              |
| Old Dominion Freight Line, Inc. (NasdaqGS:ODFL)           | INDUSTRIALS      | -0.0002 | 1.1872      | 1.6627          | 0.5506         | 0.0699                     | 48.07%            | 6.99%             | 44.94%              |
| W. R. Berkley Corporation (NYSE:WRB)                      | FINANCIALS       | 0.0004  | 0.9140      | 1.5946          | 0.3742         | 0.0647                     | 30.95%            | 6.47%             | 62.58%              |
| Viatris Inc. (NasdaqGS:VTRS)                              | HEALTHCARE       | -0.0005 | 1.6751      | 2.1707          | 0.5850         | 0.0634                     | 52.16%            | 6.34%             | 41.50%              |
| CSX Corporation (NasdaqGS:CSX)                            | INDUSTRIALS      | 0.0001  | 1.2083      | 1.5356          | 0.5439         | 0.0584                     | 48.55%            | 5.84%             | 45.61%              |
| The Allstate Corporation (NYSE:ALL)                       | FINANCIALS       | -0.0006 | 1.4642      | 1.8879          | 0.4795         | 0.0483                     | 43.12%            | 4.83%             | 52.05%              |
| The Hartford Insurance Group, Inc. (NYSE:HIG)             | FINANCIALS       | 0.0001  | 1.3754      | 1.9443          | 0.4071         | 0.0481                     | 35.90%            | 4.81%             | 59.29%              |
| T. Rowe Price Group, Inc. (NasdaqGS:TROW)                 | FINANCIALS       | 0.0000  | 1.1230      | 1.3168          | 0.4901         | 0.0428                     | 44.73%            | 4.28%             | 50.99%              |
| Sysco Corporation (NYSE:SYV)                              | CONSUMER STAPLES | -0.0003 | 0.6794      | 0.8152          | 0.3893         | 0.0422                     | 34.71%            | 4.22%             | 61.07%              |
| EQT Corporation (NYSE:EQT)                                | ENERGY           | -0.0003 | 1.2805      | 1.1299          | 0.4467         | 0.0409                     | 40.58%            | 4.09%             | 55.33%              |
| Paychex, Inc. (NasdaqGS:PAYX)                             | INDUSTRIALS      | -0.0002 | 1.3770      | 1.4073          | 0.5524         | 0.0400                     | 51.24%            | 4.00%             | 44.76%              |

Source: author's elaboration using MATLAB outputs.

Table 4.1.1 summarizes the stocks displaying the largest  $R^2$  values, alongside their estimated market beta, sector beta, and the breakdown of total variance into market, sector, and residual risk components, the findings show how the MIM changes the interpretation of return dynamics through the explicit incorporation of sector effects. The most pronounced gains in explanatory power are observed

among firms primarily operating in the Healthcare, Industrials, and Financials sectors; in particular Charles River Laboratories International achieves the largest increase in explanatory capacity, with  $\Delta R^2$  of 0.1215 and a total  $R^2$  that rises from 0.4363 under the SIM to 0.5578 under the MIM, Johnson & Johnson likewise records a substantial improvement, with a  $\Delta R^2$  of 0.1070, while comparable increases are also evident for companies such as Cencora, Biogen, IDEXX Corporation, and Old Dominion Freight Line (Elton et al., 2014; Fama & French, 2015); the table highlighted the substantial magnitude of the estimated sector betas: Charles River Laboratories, for example, reports a sector beta of 2.2426, while Johnson & Johnson and Viatris similarly exhibit sector sensitivities exceeding 2, these evidences imply that, for these firms, sector-specific factors explain part of return variability beyond the overall market factor, therefore including orthogonalized sector indices improve the model (Elton et al., 2014).

The variance decomposition further confirms this interpretation: for Charles River Laboratories, the sector component represents around 12.15% of the total variance, whereas the market factor accounts for approximately 43.63%, the remaining 44.22% is attributable to residual, idiosyncratic risk; a similar result is observed for Johnson & Johnson, for which sectoral systematic variation explains more than 10% of the overall variance, while residual risk continues to be economically relevant for most securities, the presence of the sector factor nonetheless lowers the proportion of variability not captured by the one-factor model (Elton et al., 2014; Fama & French, 2015).

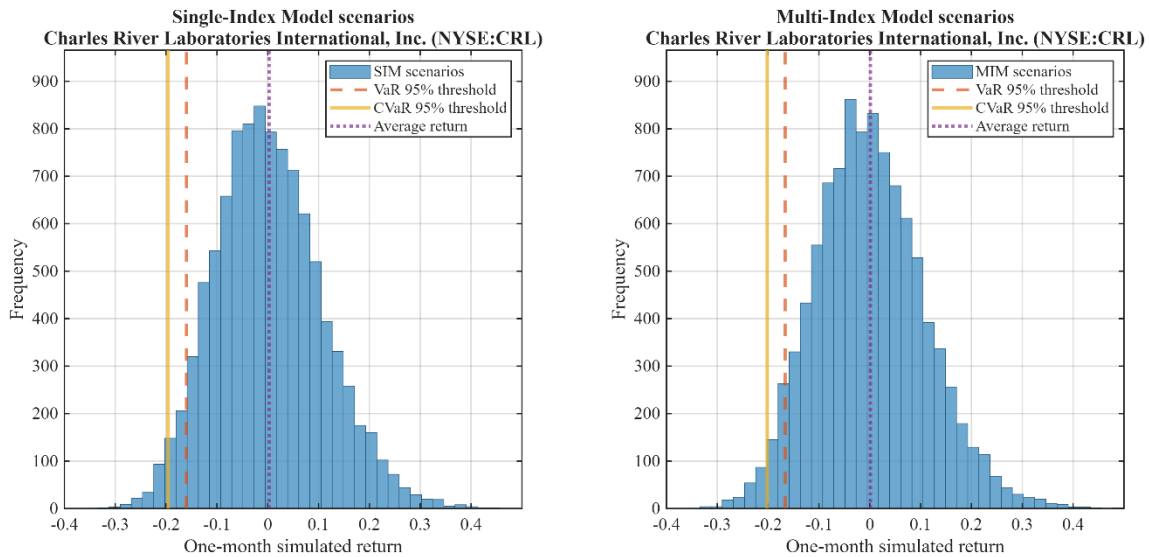
Overall, these results align with the implications drawn in Chapter 4, illustrating the persistent influence of sector dependence on equity returns (James et al., 2022).

#### 4.1.2 Stock-Level Scenario Comparison: SIM vs MIM

The more elaborate dependence structure of the Multi-Index Model can be examined by comparing the simulated return distributions generated by the SIM and MIM for a representative stock; given its substantially superior explanatory capacity, Charles River Laboratories International was selected as the case study:

the firm exhibits the highest  $\Delta R^2$  within the sample, along with a distinct sector beta linked to the Healthcare industry, therefore, the company constitutes a suitable example for assessing how the inclusion of sectoral components modifies the composition of simulated returns (Elton et al., 2014; Tulloch et al., 2020).

*Figure 4.1.1 - Comparison of SIM and MIM Simulated Return Distributions for Charles River Laboratories International*



*Source: author's elaboration using MATLAB outputs.*

The two histograms illustrate the empirical distributions of one-month simulated returns generated under the Single-Index and Multi-Index models, respectively; in each case, the plots show the frequency of the simulated outcomes, along with the associated average return, VaR threshold, and CVaR threshold at the 95% confidence level (Fairbrother et al., 2022; Rockafellar & Uryasev, 2000). Initially, the two distributions appear to follow a largely similar overall pattern, characterized by moderate positive skewness and a pronounced left tail, nonetheless, a more detailed examination highlights some discrepancies between the two scenario-generation methods (Staino & Russo, 2020; Fairbrother et al., 2022): under the SIM, the simulated return distribution is more concentrated around the centre, whereas the MIM distribution is more dispersed and exhibits a more distinctly structured tail, consistent with the inclusion of an extra systematic source of variability beyond the common aggregate market component (the

orthogonalized sector factor), showing that the MIM captures the return-generation process more realistically (Elton et al., 2014).

*Table 4.1.2 - Summary Statistics of SIM and MIM Simulated Return Distributions for Charles River Laboratories International*

| Model      | Mean  | Volatility | VaR     | CVaR    | Min     | Max    |
|------------|-------|------------|---------|---------|---------|--------|
| <b>SIM</b> | 0,25% | 10,60%     | -15,99% | -19,58% | -35,16% | 47,96% |
| <b>MIM</b> | 0,12% | 10,78%     | -16,64% | -20,17% | -34,28% | 48,41% |

*Source: author's elaboration using MATLAB outputs.*

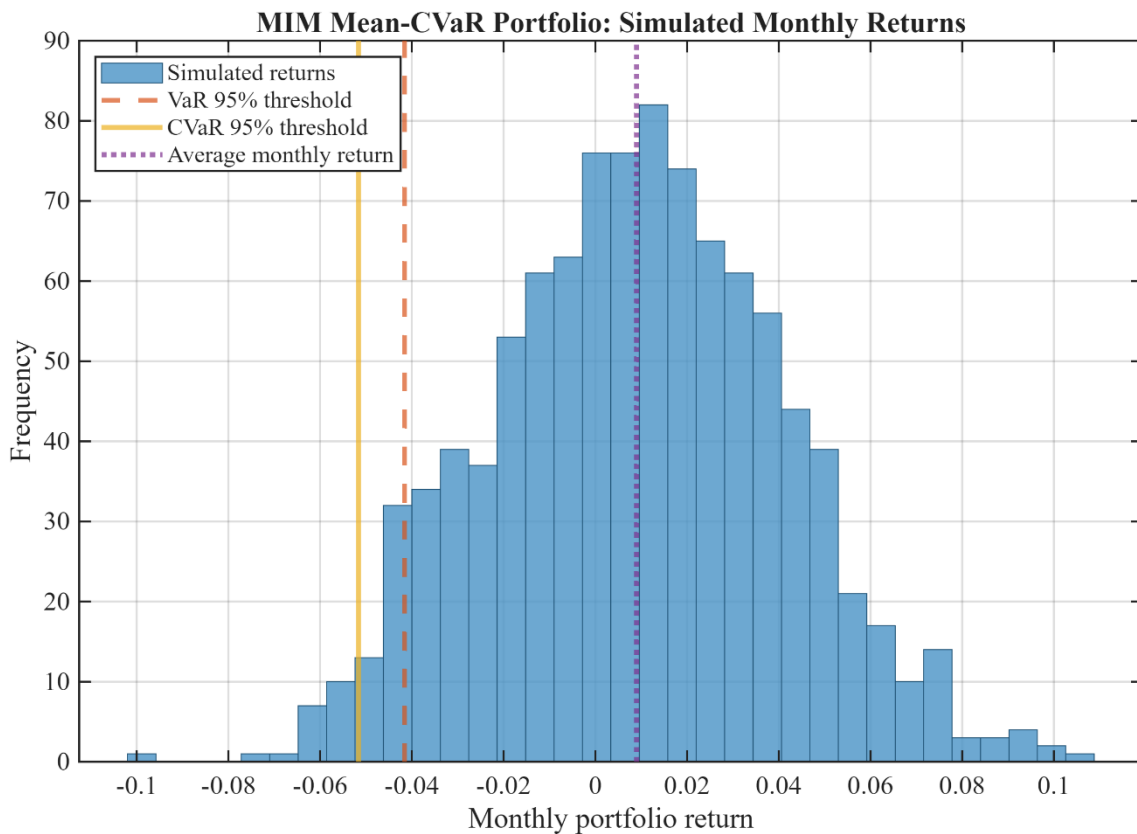
The summary statistics reported in table 4.1.2 substantiate these differences in quantitative terms: within the SIM, the mean simulated monthly return is close to 0.25%, while the MIM yields a slightly lower average of approximately 0.12%; while, volatility increases marginally from 0.1060 to 0.1078, reflecting the fact that sector-level effects result in a slightly broader range of possible outcomes (Elton et al., 2014). The downside-risk measures follow a similar trajectory: under the SIM, the 95% VaR is approximately  $-15.99\%$ , whereas under the MIM it declines to  $-16.64\%$ ; correspondingly, the CVaR worsens from  $-19.58\%$  to  $-20.17\%$ ; although differences are limited in size, they suggest that the MIM yields moderately more adverse outcomes in the lower tail for this asset (Rockafellar & Uryasev, 2000). The behaviour of the extreme tails also deserves attention: under the MIM, the lowest simulated return remains similar to that obtained under the SIM, while the highest return increases modestly to around 48.41%, meaning that the Multi-Index approach allows for a wider range of positive outcomes, resulting in greater overall dispersion in simulated returns (Elton et al., 2014; Fairbrother et al., 2022).

Methodologically, the comparison shows that the MIM can produce different simulated distributions, even when average expected returns remain similar, because by accounting for sector dependence, the optimization process captures the risk structure more accurately, particularly in relation to downside-risk dynamics and the correlation of firms exposed to comparable economic conditions (Elton et al., 2014; Staino & Russo, 2020).

#### 4.1.3 Simulated Portfolio Return Distribution under the MIM

The study can be now extended to the portfolio perspective by analysing the empirical distribution of simulated monthly portfolio returns produced within the MIM; the scenario generation procedure integrates the estimated market and sector components across all securities within the investment universe, producing a consistent set of portfolio return outcomes: the resulting distribution reflects overall market exposure, movements shared within specific sectors and the diversification mechanisms inherent in the portfolio construction process (Elton et al., 2014; James et al., 2022).

*Figure 4.1.2 - Distribution of Simulated Monthly Returns for the MIM Mean-CVaR Portfolio*



*Source: author's elaboration using MATLAB outputs.*

Figure 4.1.2 illustrates the empirical distribution of simulated monthly portfolio returns generated under the Multi-Index Mean-CVaR approach, specifically, the blue bars indicate the relative frequency of the simulated outcomes, whereas the vertical reference lines mark the mean simulated return alongside the 95% VaR

and CVaR limits (Rockafellar & Uryasev, 2000; Staino & Russo, 2020); the distribution is centred on moderately positive returns, with an average monthly return slightly above 1%, but the clear left tail shows that the model still assigns a significant probability to negative market conditions (Rockafellar & Uryasev, 2000; Fairbrother et al., 2022). The position of the VaR and CVaR thresholds provides additional insight into the downside risk shown by the simulated scenarios: the VaR threshold is situated at approximately  $-4\%$ , whereas the CVaR threshold is positioned further in the left tail, near  $-5\%$ , indicating that losses become more severe once the portfolio is exposed to extreme tail outcomes, in line with the Mean–CVaR methodology, the optimization considers both the probability of negative returns and the average loss in the worst part of the return distribution (Rockafellar & Uryasev, 2000; Staino & Russo, 2020).

The simulated results are less dispersed at the portfolio level: compared with the stock outcomes discussed earlier, the portfolio returns are more concentrated around the centre of the distribution, a result consistent with the diversification mechanism in portfolio formation, since investing in firms from different sectors can reduce some of the idiosyncratic fluctuations in stock returns (James et al., 2022; Liu et al., 2018); however, the left tail remains visible, indicating that diversification reduces but does not eliminate downside risk, as common sector shocks can still affect several firms simultaneously and expose the portfolio to adverse events (Elton et al., 2014; James et al., 2022).

These simulation results are then used in the Mean–CVaR optimization to select portfolio weights based on expected returns and the risk of extreme losses.

#### 4.2 Mean–CVaR Portfolio Construction under the Multi-Index Model

This subsection examines the portfolio allocation obtained using the Multi-Index Mean–CVaR approach, since Chapter 3 has already presented the formal Mean–CVaR optimization problem, its mathematical derivation is not repeated here, instead, it concentrates on the structure and economic meaning of the optimized portfolio under the MIM; therefore, the subsequent discussion focuses on the

portfolio weights, the sector allocation implied by the optimization results, and the portfolio's aggregate risk profile (Elton et al., 2014; Fama & French, 2015); the analysis focuses especially on how the dependence structure in the MIM scenarios influences the final distribution of allocations across individual securities and sectors.

#### 4.2.1 Optimal Portfolio Weights

*Table 4.2.1 - Optimal Portfolio Weights under the MIM Mean-CVaR Framework*

| Stock                                             | GICS Sector            | Weight |
|---------------------------------------------------|------------------------|--------|
| Eastman Chemical Company (NYSE:EMN)               | MATERIALS              | 0,1297 |
| NiSource Inc. (NYSE:NI)                           | UTILITIES              | 0,1162 |
| Hasbro, Inc. (NasdaqGS:HAS)                       | CONSUMER DISCRETIONARY | 0,1157 |
| L3Harris Technologies, Inc. (NYSE:LHX)            | INDUSTRIALS            | 0,1037 |
| M&T Bank Corporation (NYSE:MTB)                   | FINANCIALS             | 0,0902 |
| IDEXX Laboratories, Inc. (NasdaqGS:IDXX)          | HEALTHCARE             | 0,0864 |
| PACCAR Inc (NasdaqGS:PCAR)                        | INDUSTRIALS            | 0,0811 |
| Albemarle Corporation (NYSE:ALB)                  | MATERIALS              | 0,0469 |
| The PNC Financial Services Group, Inc. (NYSE:PNC) | FINANCIALS             | 0,0430 |
| Cisco Systems, Inc. (NasdaqGS:CSCO)               | INFORMATION TECHNOLOGY | 0,0351 |
| Hologic, Inc. (NasdaqGS:HOLX)                     | HEALTHCARE             | 0,0340 |
| SBA Communications Corporation (NasdaqGS:SBAC)    | REAL ESTATE            | 0,0289 |
| UDR, Inc. (NYSE:UDR)                              | REAL ESTATE            | 0,0211 |
| Intel Corporation (NasdaqGS:INTC)                 | INFORMATION TECHNOLOGY | 0,0207 |
| Camden Property Trust (NYSE:CPT)                  | REAL ESTATE            | 0,0156 |
| Walgreens Boots Alliance, Inc. (NasdaqGS:WBA)     | CONSUMER STAPLES       | 0,0125 |
| Fair Isaac Corporation (NYSE:FICO)                | INFORMATION TECHNOLOGY | 0,0078 |
| Norfolk Southern Corporation (NYSE:NSC)           | INDUSTRIALS            | 0,0061 |
| Eaton Corporation plc (NYSE:ETN)                  | INDUSTRIALS            | 0,0026 |
| Baxter International Inc. (NYSE:BAX)              | HEALTHCARE             | 0,0024 |

*Source: author's elaboration using MATLAB outputs.*

To define the optimal portfolio weights, table 4.2.1 summarizes the active positions selected through the optimization process, including their sector classifications and corresponding portfolio weights, precisely, the allocation shows that the portfolio is relatively concentrated, with a small number of securities representing a significant share of the total portfolio: the largest portfolio weight is allocated to Eastman Chemical Company in the Materials sector, accounting for approximately 12.97% of the total, while other significant positions include NiSource in the Utilities sector (11.62%), Hasbro in Consumer Discretionary (11.57%), L3Harris Technologies in Industrials (10.37%), and M&T Bank Corporation in Financials (9.02%); in addition, considerable weights are observed

for IDEXX Laboratories within Healthcare and PACCAR within Industrials, both exceeding 8% (Rockafellar & Uryasev, 2000). An important characteristic of the allocation is the diversification of its leading positions across different sectors: the portfolio gives the highest weights to companies exposed to different types of systematic and sectoral risks, in line with the economic rationale of the MIM (Elton et al., 2014; Fama & French, 2015); it is also important to note that position weights gradually decrease after the initial group of large allocations, meaning that although the largest holdings have weights above 10%, several smaller positions such as Fair Isaac Corporation, Norfolk Southern Corporation, Eaton Corporation, and Baxter International, make only a negligible contribution to overall portfolio; this configuration shows that the optimization mainly relies on a small number of securities to determine the portfolio's risk and return, while the other positions are included mainly to improve diversification and rebalance risk (Fulkerson & Riley, 2019; Ferreira et al., 2021;).

Overall, the allocation balances concentration and diversification, assigning greater weight to a limited number of securities with favourable downside characteristics while maintaining exposure to different sectors and firms, reducing dependence on any single asset or industry.

#### 4.2.2 Sector Allocation of the MIM Portfolio

A more comprehensive understanding of the portfolio composition can be achieved by aggregating the individual holdings at the GICS sector level, in this regard, table 4.2.2 reports the total portfolio weight attributed to each sector following the optimization process.

Table 4.2.2 - Sector Allocation of the Optimized MIM Mean–CVaR Portfolio

| Sector                        | Weight |
|-------------------------------|--------|
| <b>INDUSTRIALS</b>            | 19,36% |
| <b>MATERIALS</b>              | 17,67% |
| <b>FINANCIALS</b>             | 13,32% |
| <b>HEALTHCARE</b>             | 12,28% |
| <b>UTILITIES</b>              | 11,62% |
| <b>CONSUMER DISCRETIONARY</b> | 11,57% |
| <b>REAL ESTATE</b>            | 6,56%  |
| <b>INFORMATION TECHNOLOGY</b> | 6,37%  |
| <b>CONSUMER STAPLES</b>       | 1,25%  |
| <b>ENERGY</b>                 | 0,00%  |
| <b>COMMUNICATION SERVICES</b> | 0,00%  |

Source: author's elaboration using MATLAB outputs.

The allocation results indicate that Industrials is the largest sector holding in the portfolio, with a weight of approximately 19.36% of the total, Materials accounts for a slightly lower proportion at around 17.67%, whereas Financials and Healthcare contribute roughly 13.32% and 12.28%, respectively; a meaningful weight is also allocated to Utilities and Consumer Discretionary, each representing more than 11% of the portfolio. At the same time several sectors are assigned relatively small weights: Information Technology comprises about 6.37%, while Consumer Staples is limited to 1.25%; notably, Energy and Communication Services receive no allocation in the optimized portfolio.

Economically, the results show that the Mean–CVaR optimization model within the MIM assigns higher weights to sectors with more favourable downside characteristics, in particular, Industrials, Materials, and Utilities generally combine relatively high expected returns with lower exposure to losses in the lower tail during the estimation period: this behaviour is consistent with the Mean–CVaR criterion, which aims to improve expected returns while reducing losses in the worst scenarios (Rockafellar & Uryasev, 2000, Elton et al., 2014). By contrast, the complete lack of allocations to Energy and Communication Services implies that in the scenarios generated by the MIM, these sectors contribute significantly to portfolio downside risk, their effect on extreme losses is large enough to outweigh

the benefits of diversification (James et al., 2022). The optimized portfolio also maintains a reasonable level of sectoral diversification: the allocation remains spread across several industries, only some sectors receive greater weights, which is particularly relevant to the Multi-Index context, which accounts for common movements within sectors and limits concentration in firms exposed to similar macroeconomic risks (Fama & French, 2015; James et al., 2022).

Finally, making a comparison with the sector weights derived from the SIM optimization reported in Chapter 3 it's possible to notice that the MIM portfolio is more balanced across cyclical, defensive, and sectoral components, while the SIM portfolio is more concentrated in sectors such as Industrials, Consumer Staples, and Information Technology; this difference reflects the fact that the SIM captured dependence among securities only through the common market factor (Elton et al., 2014).

#### 4.2.3 Risk Characteristics of the Optimized Portfolio

In addition to examining the composition of individual weights and sector allocations, the optimized portfolio is further assessed using the overall risk and allocation metrics analysed in the following table 4.2.3.

*Table 4.2.3 - Risk Characteristics of the Optimized MIM Mean–CVaR Portfolio*

| MIM MEAN CVAR PORTFOLIO           | Values |
|-----------------------------------|--------|
| <b>Expected monthly return</b>    | 0,90%  |
| <b>VaR loss 95%:</b>              | 4,16%  |
| <b>CVaR loss 95%:</b>             | 5,16%  |
| <b>Sum of weights</b>             | 1      |
| <b>Number of active positions</b> | 20     |
| <b>Max weight</b>                 | 12,97% |

*Source: author's elaboration using MATLAB outputs.*

The portfolio generated by the Multi-Index Mean–CVaR model has an estimated monthly return of approximately 0.90%, suggesting that the model can still select assets with favourable expected returns while limiting downside risk; meanwhile, the 95% VaR and CVaR estimates, at 4.16% and 5.16%, respectively, point to relatively limited downside risk in the simulated scenarios, and the small

difference between VaR and CVaR also indicates that losses do not become much more severe when focusing on the worst outcomes (Rockafellar & Uryasev, 2000; Fairbrother et al., 2022). The table also shows that the portfolio is fully invested, with weights adding up to one, the optimization identifies 20 active positions, with the largest allocation accounting for approximately 12.97%; the optimized portfolio serves as the foundation for the analysis presented in the subsequent section.

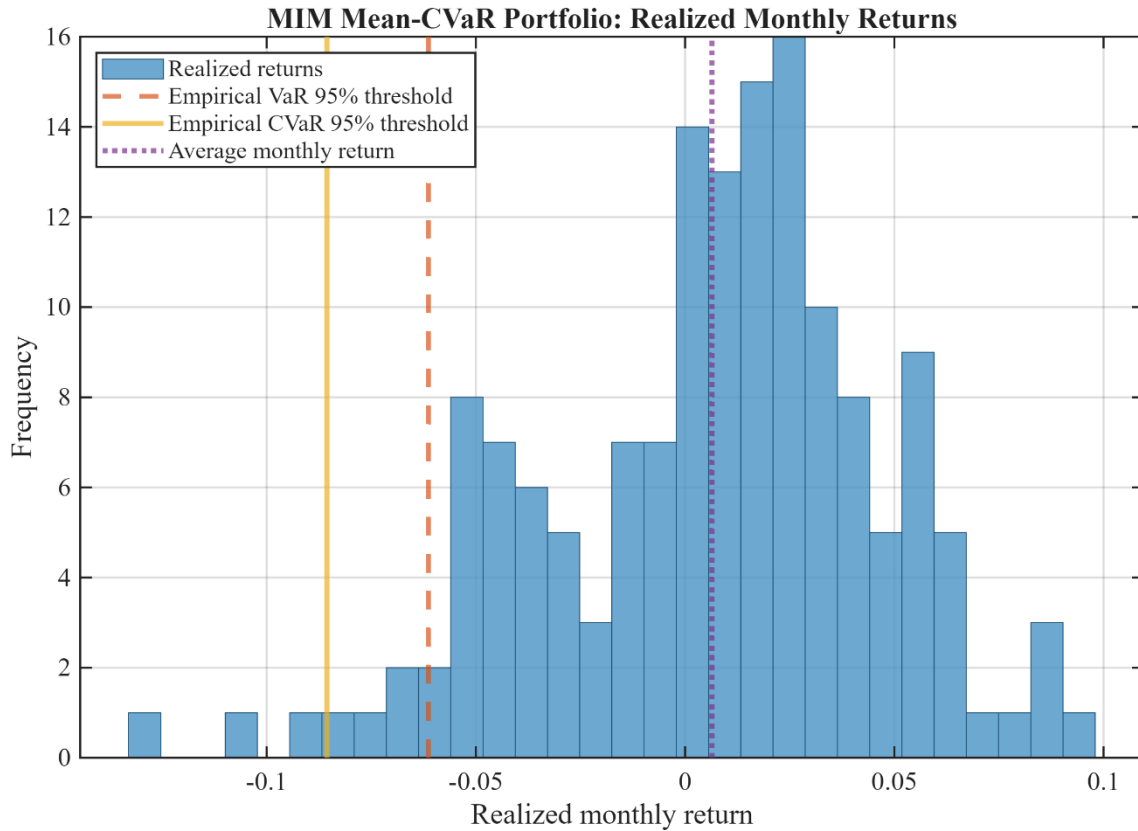
### 4.3 Realized Performance Evaluation

The final phase involves assessing the strategy's performance across time: this step is crucial, as it moves the evaluation from the simulated setting to actual market outcomes, thereby enabling the effectiveness of the MIM allocation to be tested throughout the entire backtesting period.

#### 4.3.1 Realized Monthly Return Distribution of the MIM Strategy

This stage examines how the optimized allocation performed when subjected to the observed dynamics of the market: in this regard, comparing simulated and realized returns is particularly informative, as it enables the assessment of the consistency of the simulation process in terms of how portfolio performance evolved over time.

Figure 4.3.1 - Distribution of Realized Monthly Returns for the MIM Mean-CVaR Strategy



Source: author's elaboration using MATLAB outputs.

Figure 4.3.1 illustrates the empirical distribution of the monthly returns realized by the MIM portfolio across the entire backtesting window, more precisely the histogram is obtained by pooling the realized monthly returns recorded after each portfolio rebalancing date; the vertical lines report, for comparison, the sample mean realized monthly return and the empirical 95% VaR and CVaR levels derived directly from the observed return realizations, specifically the blue bars denote the empirical frequencies of the realized monthly outcomes, meaning that, the height of each blue bar reflects the number of realized monthly observations within the corresponding return interval; for example, the highest bars indicate monthly returns near 2%, suggesting that portfolio outcomes of this magnitude occurred more often throughout the backtesting period, relative to rarer extreme positive or negative results; the orange dashed line marks the empirical VaR threshold, the yellow solid line indicates the empirical CVaR value, and the purple dotted line corresponds to the average realized monthly return (Rockafellar & Uryasev, 2000;

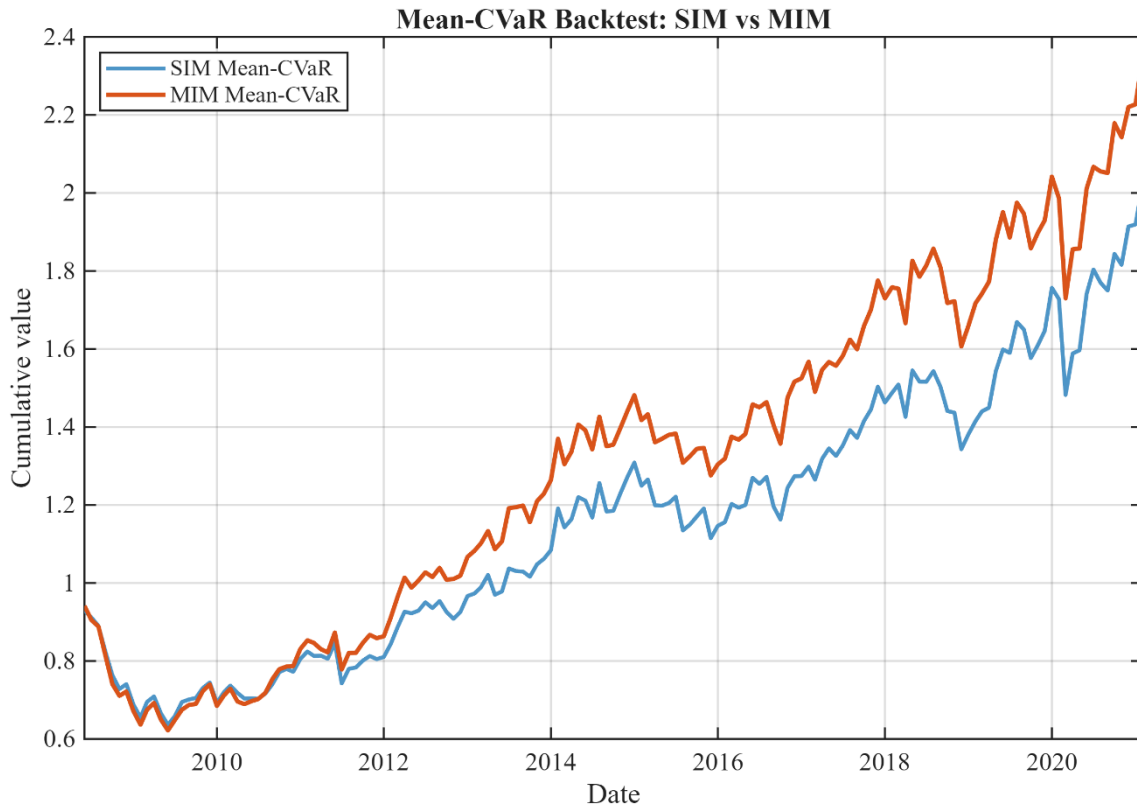
Staino & Russo, 2020).

Observing the histogram it is possible to notice that the realized distribution is mainly concentrated around moderately positive monthly returns, with the mean realized outcome slightly above zero, while the left tail remains clearly present; compared with the smoother distribution of simulated portfolio returns shown in Section 5.1.3, the realized histogram is less regular, more widely dispersed, and includes several distinct negative outcomes (Rockafellar & Uryasev, 2000; Fairbrother et al., 2022). Such distinction can be explained economically: simulated scenarios are generated under strict assumptions based on the estimated factor structure, while realized returns are also affected by sudden market disruptions, periods of high volatility, and unexpected shocks affecting multiple securities that cannot be fully anticipated (Staino & Russo, 2020; Fairbrother et al., 2022); in both scenarios, most observations are concentrated around slightly positive returns, while large losses appear only in a small part of the distribution, the lower tail remains relevant however, as shown by the empirical VaR and CVaR thresholds, even though the optimization strategy is designed to minimize CVaR. (Rockafellar & Uryasev, 2000).

#### 4.3.2 Cumulative Performance and Drawdown Comparison: SIM vs MIM

To further examine the dynamic evolution of portfolio performance across the full investment horizon, it is helpful to compare the cumulative value trajectories and drawdown dynamics of the SIM and MIM Mean–CVaR strategies, thereby evaluating how profits and losses are progressively realized over time under different market regimes.

Figure 4.3.2 - Cumulative Value Comparison between the SIM and MIM Mean-CVaR Portfolios



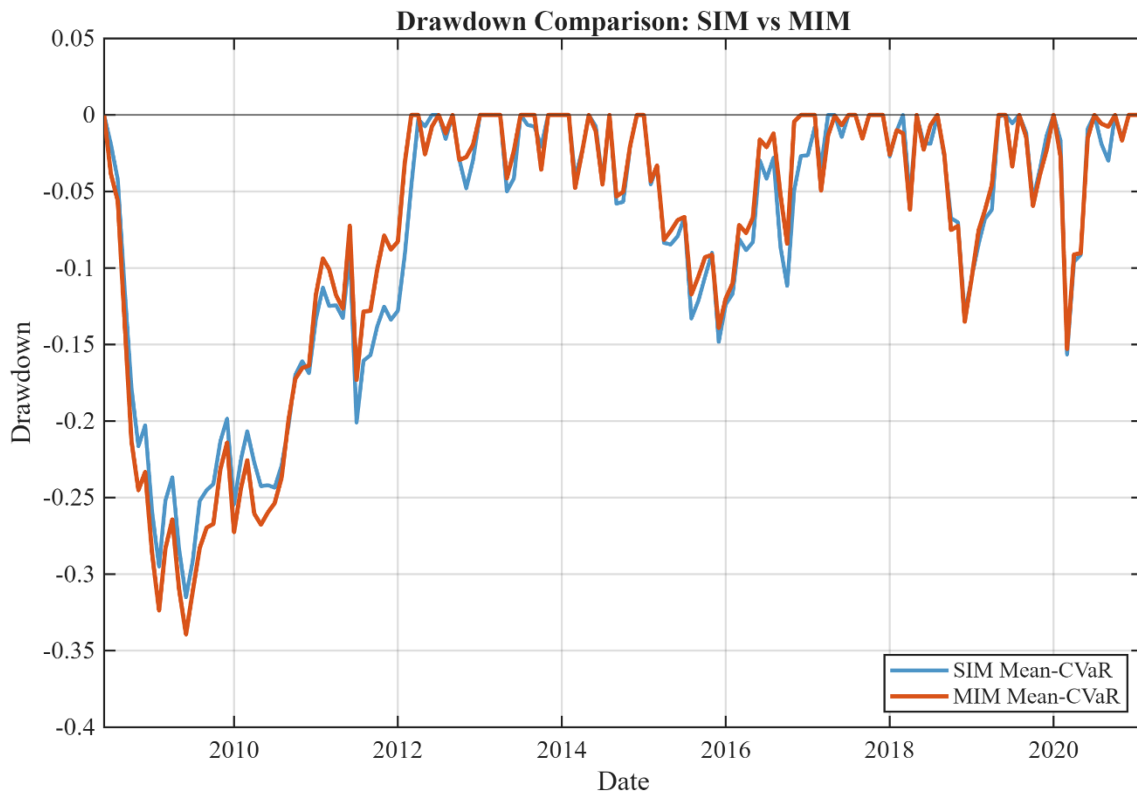
Source: author's elaboration using MATLAB outputs.

From a methodological point of view, the cumulative-value series reported in figure 4.3.2 are constructed by repeatedly compounding the realized monthly portfolio returns obtained after each rebalancing date: each point on the curve shows how one monetary unit invested at the beginning of the backtesting period would have performed over time under the respective strategy; for instance, a cumulative value of about 1.50 implies that the portfolio has produced an overall gain of approximately 50% relative to the initial investment, whereas values below 1 correspond to intervals in which any cumulative losses have not yet been fully recovered (Staino & Russo, 2020). The comparison shows that the MIM strategy generally performs better than the SIM allocation over the observation period, in particular way both portfolios experienced a significant decline at the beginning of the backtest, particularly during the 2008–2009 financial crisis, then their recovery patterns differed: from around 2012 onwards, the MIM portfolio generated returns at a faster rate and maintained a higher cumulative performance than the SIM

portfolio (Manda, 2010; Securities and Futures Commission, 2014).

In addition, the figure shows that the difference in performance remains consistent over time, as it grows gradually when one strategy achieves slightly better monthly results, which is an important finding because it shows that using specific sectoral information to generate scenarios affects both individual allocation decisions and the portfolio's overall development under different market conditions (Fama & French, 2015; Staino & Russo, 2020); such distinction becomes more evident in the latter half of the sample: around 2017–2018, for example, the MIM portfolio attains cumulative values close to 1.80, whereas the SIM strategy remains nearer to 1.50, a similar gap is also observable toward the end of the horizon, when the MIM allocation rises to cumulative levels exceeding 2.30, while the SIM portfolio stays around 2.00; in general, this behaviour suggests that an investment of one monetary unit using the MIM approach more than doubles over the sample period, whereas the SIM strategy produces a smaller increase (Staino & Russo, 2020; James et al., 2022).

Figure 4.3.3 - Drawdown Comparison between the SIM and MIM Mean-CVaR Portfolios



Source: author's elaboration using MATLAB outputs.

The drawdown comparison in figure 4.3.3 supports these findings by showing how much each strategy lost compared with its previous cumulative maximum, in other words, the drawdown series measures the percentage decline in portfolio value from its highest recorded level at that time; for example, a drawdown of  $-0.20$  implies that, at that date, the portfolio value is approximately 20% below its historical peak; the two portfolios show similar drawdown patterns in several periods, particularly during broad market crises when both are affected by common systematic factors, at the same time, the size and length of the declines differ: at the beginning of the sample period, the MIM portfolio fell by approximately 34%, while the SIM portfolio declined by about 31% (Manda, 2010; Elton et al., 2014). The drawdown paths show that the MIM strategy generally recovers faster after periods of negative performance, for example, after the 2015–2016 drawdown, the MIM portfolio returns close to zero drawdown relatively quickly, while the SIM strategy remains below its previous high for longer, a similar pattern can be

observed after the market decline in 2018–2019; overall, the Multi-Index strategy does not eliminate downside risk, but it appears to support a faster recovery after negative shocks (Mazur et al., 2021; Fairbrother et al., 2022).

Taken as a whole, the cumulative-value and drawdown results indicate that including sector systematic components in the Mean–CVaR approach leads to a different portfolio performance over time; compared with the SIM specification, the MIM strategy achieves higher cumulative growth while maintaining a similar level of downside risk, although it occasionally experiences more severe short-term losses.

#### 4.3.3 Summary Performance Comparison

To complete the empirical analysis, the results of the Mean–CVaR optimization under the Multi-Index Model are compared with those of the corresponding portfolio estimated using the Single-Index model, to focus on the main performance indicators to assess the economic implications of the two approaches.

*Table 4.3.1 - Comparative Performance Statistics: SIM Mean–CVaR vs MIM Mean–CVaR Portfolios*

| Strategy             | SIM Mean-CVaR | MIM Mean-CVaR |
|----------------------|---------------|---------------|
| TotalReturn          | 1,0239        | 1,3477        |
| AnnualizedReturn     | 0,0569        | 0,0692        |
| AnnualizedVolatility | 0,1381        | 0,1396        |
| SharpeRatio          | 0,4117        | 0,4959        |
| MaxDrawdown          | -0,3152       | -0,3393       |
| AvgMonthlyReturn     | 0,0054        | 0,0064        |
| WorstMonth           | -0,1422       | -0,1291       |
| BestMonth            | 0,0988        | 0,0964        |

*Source: author's elaboration using MATLAB outputs.*

Table 4.3.1 summarizes the key realized performance metrics for both strategies, reporting cumulative return, annualized return and volatility, Sharpe ratio, maximum drawdown, and the distributional properties of monthly realized returns; the overall results show that adding sector systematic factors to the MIM leads to higher portfolio growth and better risk-adjusted performance, at the same time its volatility remains similar to that observed under the SIM specification (Elton et

al., 2014; James et al., 2022). The MIM Mean–CVaR portfolio generated a cumulative return of approximately 1.35 over the sample period, compared with 1.02 for the SIM strategy, this means that an initial investment of one monetary unit would have grown to about 2.35 under the MIM and to approximately 2.02 under the SIM; the divergence is also visible in annualized results: the MIM portfolio delivers an annualized return of 6.92%, while the SIM portfolio remains close to 5.69%, results aligned with the cumulative-value evolution shown earlier in figure 5.3.2, in which the MIM path gradually diverges from the SIM portfolio, particularly during the latter half of the sample (Staino & Russo, 2020). This improvement in realized performance does not lead to a significant increase in overall volatility: annualized volatility remains almost unchanged, at approximately 13.96% for the MIM and 13.81% for the SIM; as a result, the higher realized returns reported by the MIM portfolio lead to a higher Sharpe ratio, which increases from 0.4117 to 0.4959.

The two approaches also differ in their performance during market downturns: the MIM portfolio records a slightly higher maximum drawdown than the SIM portfolio (–33.93% versus –31.52%); however, its monthly returns are less extreme during adverse periods, since the lowest monthly return for MIM is approximately –12.91%, compared with –14.22% for SIM, conversely, SIM achieves a slightly higher maximum monthly return (9.88% versus 9.64%). Therefore, the enhanced cumulative returns associated with the MIM arise from a steadier series of moderately positive results across the observation window (Rockafellar & Uryasev, 2000; Fairbrother et al., 2022).

To conclude, the comparative results show that extending the Mean–CVaR optimization model from the Single-Index Model to the Multi-Index Model improves portfolio performance during the backtesting period; despite difficult market conditions and several periods of losses, the MIM portfolio achieved higher cumulative returns and better risk-adjusted performance.

## 5 Benchmark comparison

### 5.1 Introduction and Benchmark Strategies

The analyses conducted in the preceding chapters addressed the development and assessment of portfolio strategies within the Mean–CVaR optimization process, first using the Single Index Model and then extending the analysis to the Multi-Index Model; the findings indicated that different specifications of the return-generating process can result in variations in portfolio composition, in the estimated level of downside risk, and, consequently, in the realized performance of the strategies. However, determining the effectiveness of an investment approach requires more than a direct comparison across alternative model specifications, then a more comprehensive evaluation should also examine how the resulting portfolios perform against other allocation methods that are widely used in practice.

Accordingly, this chapter compares the optimized Mean–CVaR portfolios with other portfolio management approaches, the comparison uses the same evaluation criteria to assess their actual performance; the benchmark set comprises both passive allocation rules and strategies based on optimization, in particular, the analysis uses the S&P 500 Index as the market benchmark, an equally weighted portfolio as a simple diversification reference, and two conventional mean–variance constructions corresponding to the Minimum Variance and Mean Variance portfolios (Markowitz, 1952; Sharpe, 1964; Yan & Zhang, 2017 ), these are subsequently compared with the Mean–CVaR portfolios introduced in the preceding chapters and with Minimum CVaR portfolios included as supplementary benchmarks focused on downside risk (Rockafellar & Uryasev, 2000). The analysis is performed on the basis of the same historical dataset and employs the rolling backtesting procedure used in the preceding chapters, guaranteeing complete methodological alignment across all portfolio strategies, so that any observed differences in performance are attributable primarily to the allocation approach (Staino & Russo, 2020; Bailey et al., 2021).

The chapter is organized as follows: first, each benchmark approach is briefly introduced, along with the reasoning behind its portfolio construction, the portfolios are then compared using various measures of performance and risk, with a clear distinction between minimum-risk and mean-risk investment targets; finally, the outcomes are interpreted collectively to determine the relative merits and limitations of the various methods and to assess whether the Mean-CVaR optimization process developed in this thesis performs better than more conventional portfolio allocation strategies.

#### 5.1.1 S&P 500

The S&P 500 is used as the passive benchmark to compare the performance of the alternative portfolio strategies; widely followed as an equity index, it is generally considered to be a proxy for the U.S. stock market and, in a broader sense, for the market portfolio postulated in modern asset pricing frameworks (Sharpe, 1964; Gregoire, 2020); in this study, the benchmark portfolio is based on the historical returns of the S&P 500 included in the dataset, unlike the optimized portfolios presented in the previous chapters, it does not rely on forecasts, risk models, or optimization methods, but it represents the return an investor would have achieved by passively holding the market over the same period (Gregoire, 2020).

From a theoretical perspective, the market portfolio is central to the Capital Asset Pricing Model (CAPM) because it represents the systematic risk shared by all securities; in this model, expected returns depend on how closely an asset moves with the market, as measured by its beta, the market portfolio can therefore be viewed as the benchmark against which active investment strategies are assessed (Sharpe, 1964). In conclusion, since the S&P 500 portfolio is not actively rebalanced or optimized, its performance reflects the risk and return of the market itself and therefore provides a useful benchmark for evaluating whether portfolio optimization leads to better investment results (Sharpe, 1964).

### 5.1.2 Equal Weight

The Equal Weight portfolio is among the most elementary and commonly adopted benchmark strategies in portfolio management, specifically the investable capital is divided equally among all securities in the defined investment universe, such that every asset is assigned an identical portfolio weight:

$$w_i = \frac{1}{N}$$

where  $N$  represents the number of assets included in the portfolio (De Miguel et al., 2009).

The equal-weight strategy does not depend on specific model assumptions, which reduces the risk of estimation errors, for this reason, it provides a useful benchmark for evaluating more advanced portfolio construction methods and more specifically, the Equal Weight portfolio reflects the idea of naïve diversification, whereby investment is distributed equally among all eligible assets instead of being concentrated according to forecasts of risk or expected return. Despite its apparent simplicity, this strategy has gained substantial scholarly interest, largely because its empirical performance is frequently comparable to that of more sophisticated optimization approaches, especially in contexts where model inputs are subject to estimation error (DeMiguel et al., 2009); the Equal Weight approach embodies a core principle of portfolio construction: diversification; allocating capital equally across a broad set of securities limits the portfolio's sensitivity to risks specific to individual firms and helps prevent undue concentration in a small number of holdings, then it serves as a clear and readily interpretable reference point for evaluating the incremental value of active allocation strategies (De Miguel et al., 2009; James et al., 2022). The Equal Weight portfolio serves as a simple benchmark in this analysis: because it does not use any specific risk-reduction technique, its performance reflects the benefits of diversification alone; comparing the optimized portfolios with this benchmark shows whether the additional complexity of portfolio optimization leads to better risk-adjusted performance and

lower downside risk.

### 5.1.3 Mean–Variance

The Mean–Variance benchmark is based on Markowitz’s classical portfolio theory. It evaluates expected returns and the covariance between asset returns to determine the portfolio weights that best balance performance and risk (Markowitz, 1952).

The portfolio is derived by solving the following optimization problem:

$$\max_w \left( \mu^T w - \frac{\lambda}{2} w^T \Sigma w \right)$$

where  $\mu$  denotes the vector of expected returns,  $\Sigma$  represents the return covariance matrix  $w$  is the vector of portfolio weights  $\lambda$  is the risk-aversion coefficient that governs the balance between expected return and variance-based risk (Markowitz, 1952); in the present study, the benchmark is formulated with a risk-aversion parameter set to  $\lambda = 6$ , thereby imposing a substantial cost on portfolio volatility while retaining a clear target for expected returns: the allocation is designed to deliver favourable expected performance, rather than pursuing volatility minimization as its sole objective (Markowitz, 1952).

Economically, the Mean–Variance model quantifies risk by considering the overall dispersion of returns and by assigning equal weight to deviations above and below the mean, a feature that distinguishes it from the Mean–CVaR methodology introduced in the preceding chapters, in which the optimization problem is explicitly shaped by downside risk; consequently, comparing the two approaches helps determine whether portfolio strategies targeted at tail losses can deliver benefits relative to conventional variance-based optimization (Markowitz, 1952). Although the Mean–Variance methodology has been widely implemented, it remains vulnerable to estimation error, especially when expected returns and the covariance matrix are derived from limited historical observations, as highlighted by Simaan et al. (2018), even minor perturbations in the estimated parameters can generate pronounced changes in the resulting portfolio weights.

#### 5.1.4 Minimum Variance

The Minimum Variance portfolio is one of the most widely studied risk-based investment strategies in portfolio theory, which aim is to determine the feasible portfolio configuration that attains the lowest possible level of variance (Markowitz, 1952; Clarke et al., 2006); the optimization problem can be expressed as follows:

$$\min_w w^T \Sigma w$$

assuming the full-investment and long-only conditions, with  $w$  indicating the portfolio weight vector and  $\Sigma$  the covariance matrix of asset returns (Markowitz, 1952). A key characteristic of this approach is that portfolio formation relies solely on the estimated covariance structure of returns, in this sense, the Minimum Variance portfolio may be interpreted as a fundamentally defensive allocation, oriented primarily toward capital preservation by reducing portfolio volatility (Clarke et al., 2006).

The empirical importance of minimum-variance investing is well established in the academic literature, in particular, Clarke, de Silva and Thorley find that minimum-variance portfolios often produce risk-returns performance that are comparable to those of market benchmarks, even though their main purpose is to reduce risk; their findings helped support what is now commonly known as the low-volatility anomaly, which suggests that portfolios made up of less risky assets can generate returns that are not necessarily lower, in relative terms, than those achieved by the market as a whole (Clarke et al., 2006). Another advantage of the Minimum Variance approach is its relatively limited reliance on assumptions regarding expected returns: since portfolio weights are derived solely from the covariance matrix, the implied allocations are generally less sensitive to estimation errors than those produced by standard mean–variance methods; this robustness is particularly relevant in empirical research, where expected returns are typically observed with considerable noise and therefore remain difficult to estimate with precision (Simaan et al., 2018).

For these reasons, the Minimum Variance portfolio represents a useful benchmark for the present analysis, allowing us to assess whether approaches based on downside risk, such as the Minimum CVaR portfolio discussed in the following section, can provide additional benefits beyond those obtained by minimizing volatility alone.

#### 5.1.5 Minimum CVaR

The Minimum CVaR portfolio extends traditional portfolio optimization by substituting variance with Conditional Value-at-Risk (CVaR) as the optimization criterion: CVaR is concerned solely with the worst realizations, offering a more direct measure of tail risk, therefore the Minimum CVaR portfolio may be interpreted as a downside-risk alternative to the Minimum Variance benchmark presented in the previous section (Rockafellar & Uryasev, 2000); the portfolio is determined by minimizing the expected loss tied to the lower tail of the return distribution, in line with the formulation introduced by Rockafellar and Uryasev, the corresponding optimization problem can be then stated as follows:

$$\min_{w,t,u} \left( t + \frac{1}{(1-\alpha)S} \sum_{s=1}^S u_s \right)$$

where  $t$  denotes the Value-at-Risk threshold,  $u_s$  is the excess loss under scenario  $s$ ,  $S$  represents the total number of simulated scenarios, and  $\alpha$  specifies the confidence level that determines the distribution's tail region; precisely, CVaR is computed at a 95% confidence level, so the optimization is directed toward the worst 5% of portfolio outcomes (Rockafellar & Uryasev, 2000).

In contrast to the Mean–CVaR approach introduced in the preceding chapters, the Minimum CVaR portfolio framework does not include an explicit target return: its only objective is to determine the asset weights that minimize expected losses under adverse market scenarios, then the resulting portfolio is expected to display a markedly defensive orientation (Rockafellar & Uryasev, 2000).

This method is based on the fact that financial returns often have asymmetric distributions and extreme values, then by focusing on the worst portfolio

outcomes, CVaR shows the potential losses investors may experience once the VaR threshold has been exceeded; this characteristic makes CVaR especially suitable for risk management applications and portfolio construction when market conditions are unfavourable (Rockafellar & Uryasev, 2000).

## 5.2 Comparative analysis of portfolio strategies

The following discussion examines the portfolio strategies presented above from two complementary perspectives: the mean-risk comparison evaluates their performance by considering returns in relation to the associated level of risk, whereas the minimum-risk comparison concentrates on strategies whose primary objective is risk minimization; this distinction establishes a basis for analysing how the different approaches perform in practice when subjected to alternative optimization criteria.

### 5.2.1 Mean-Risk Comparison

*Table 5.2.1 - Performance Comparison of Mean-Risk Portfolio Strategies*

| Strategy            | Mean-CVaR SIM | Mean-CVaR MIM | Mean-Variance | Equal Weight | S&P 500 |
|---------------------|---------------|---------------|---------------|--------------|---------|
| Mean monthly return | 0,54%         | 0,64%         | 0,64%         | 0,64%        | 0,60%   |
| Monthly variance    | 0,16%         | 0,16%         | 0,24%         | 0,34%        | 0,27%   |
| VaR95_Loss          | 6,93%         | 6,13%         | 8,35%         | 9,91%        | 8,46%   |
| CVaR95_Loss         | 8,69%         | 8,56%         | 12,29%        | 15,80%       | 13,66%  |
| Max Drawdown        | -31,52%       | -33,93%       | -43,55%       | -54,25%      | -51,37% |
| Sharpe Ratio        | 0,41          | 0,50          | 0,38          | 0,28         | 0,32    |
| Skewness            | -0,63         | -0,52         | -0,93         | -0,67        | -0,77   |
| Kurtosis            | 3,9263        | 3,3292        | 4,4236        | 7,9945       | 6,8710  |
| Sortino Ratio       | 0,56          | 0,73          | 0,45          | 0,31         | 0,36    |
| Calmar Ratio        | 0,18          | 0,20          | 0,15          | 0,11         | 0,11    |
| Average Turnover    | 0,5521        | 0,5527        | 0,4325        | /            | /       |
| Jensen Alpha        | 3,08%         | 4,20%         | 2,30%         | -0,25%       | /       |
| Annual return       | 5,69%         | 6,92%         | 6,37%         | 5,71%        | 5,66%   |
| Annual volatility   | 13,81%        | 13,96%        | 16,98%        | 20,21%       | 17,91%  |

Source: author's elaboration using MATLAB outputs.

The extent to which the proposed approaches differ from more conventional portfolio allocation methods is assessed in table 5.2.1, which summarizes a set of realized performance metrics for the two Mean–CVaR portfolios, together with the Mean–Variance portfolio, the Equal Weight benchmark, and the S&P 500 index; since each strategy seeks to balance returns and risk, the analysis adopts a mean–risk perspective, in which performance is assessed in relation to both the level and the type of risk involved. At first sight, the findings indicate that the annually realized returns of all actively managed portfolios are broadly similar, ranging

from approximately 5.7% to 6.9%, instead the highest annual performance is observed for the MIM Mean–CVaR portfolio (6.92%), followed by the Mean–Variance approach (6.37%), whereas the SIM Mean–CVaR portfolio delivers a return of 5.69%; while these gaps may seem relatively small in absolute terms, their significance becomes clearer once the associated risk metrics are considered jointly.

The first key metric is annual volatility, which measures how much portfolio returns vary over time, although it is commonly used to assess risk, it does not distinguish between positive and negative changes (Clarke et al., 2006); in this regard, the Mean–Variance portfolio achieves an annual volatility of 16.98%, which is higher than the Mean–CVaR portfolios, both of which remain around 14%, while the Equal Weight benchmark shows the highest volatility among the alternatives (20.21%), while the S&P 500 registers 17.91%; these results indicate that the higher returns achieved by the Mean–Variance approach are accompanied by greater fluctuations in portfolio value.

A more detailed evaluation is provided by the Sharpe Ratio, which measures excess return generated per unit of overall risk: across the considered strategies, the MIM Mean–CVaR portfolio records the highest Sharpe Ratio (0.4959), followed by the SIM Mean–CVaR portfolio (0.4117) and the Mean–Variance benchmark (0.3752); these findings indicate that the Mean–Variance portfolio's superior expected returns do not offset its comparatively greater volatility, whereas the MIM Mean–CVaR framework translates each unit of risk into realized performance more effectively (Sharpe, 1964).

Since managing downside risk is central in our analysis, tail-risk metrics are particularly relevant: the 95% Value-at-Risk (VaR) identifies the loss level expected to be surpassed only by the worst 5% of observations, whereas the 95% Conditional Value-at-Risk (CVaR) captures the mean loss conditional on exceeding that threshold; looking at the data, the MIM Mean–CVaR portfolio yields the smallest losses, recording the lowest VaR (6.13%) and CVaR (8.56%) across the set of portfolios and the SIM Mean–CVaR approach performs similarly,

with VaR and CVaR losses of 6.93% and 8.69%, respectively; in comparison, the Mean–Variance portfolio displays more pronounced tail risk, with a VaR of 8.35% and a CVaR of 12.29%, this worsening continues for the Equal Weight benchmark, for which the CVaR increases to 15.80% (Rockafellar & Uryasev, 2000; Elton et al., 2014). These outcomes are particularly significant because they confirm that the Mean–CVaR portfolios attain the intended goal: the Mean–Variance portfolio remains competitive in terms of return generation, but the Mean–CVaR strategies offer substantially better resilience to extreme losses, furthermore, the reduction in tail risk is not achieved at the cost of performance, because the SIM Mean–CVaR portfolio generates an annual return consistent with those of the Equal Weight and S&P 500 benchmarks, whereas the MIM Mean–CVaR portfolio delivers the highest annual return among all evaluated strategies. A similar result emerges from the Sortino Ratio, which, unlike the Sharpe Ratio, accounts only for downside volatility, in this case, the MIM Mean–CVaR portfolio again records the top performance (0.7324), substantially exceeding the SIM Mean–CVaR portfolio (0.5576), the Mean–Variance portfolio (0.4538), and the benchmark alternatives; in addition, because the Sortino Ratio measures deviations arising from adverse return movements, the evidence further substantiates the finding that the MIM Mean–CVaR approach offers the most advantageous trade-off between realized returns and downside risk (Sortino & van der Meer, 1991).

The Calmar Ratio evaluates portfolio performance in relation to the maximum drawdown, which measures the largest cumulative loss experienced by a portfolio from a previous peak and therefore indicates the extent of the decline during prolonged periods of weak performance; in this case, the MIM Mean–CVaR portfolio records a slightly higher maximum drawdown than the SIM portfolio, at –33.93% compared with –31.52%, but its stronger return performance allows it to achieve the highest Calmar Ratio, equal to 0.2040. This evidence suggests that, despite experiencing a larger decline, the MIM Mean–CVaR strategy generates a more favourable return relative to the risk associated with its maximum drawdown (Keating & Shadwick, 2002).

The positive Jensen's Alpha reported across both Mean-CVaR portfolios suggests that their returns surpass those attributable solely to market exposure: the MIM Mean-CVaR approach delivers the highest value (0.0420), followed by the SIM specification (0.0308), while the Equal Weight portfolio records a marginally negative alpha, this pattern indicates that the optimization process achieves better results through a more efficient allocation of capital. (Jensen, 1968).

The last relevant metric is portfolio turnover, which captures the degree of rebalancing over time and thus reflects the intensity of the implementation process: the Mean-CVaR portfolios show mean turnover values marginally above 0.55, relative to 0.43 for the Mean-Variance portfolio, an evidence that suggests that the downside-risk strategy requires more frequent portfolio adjustments as market conditions and estimates of tail risk change; while greater turnover may translate into higher transaction costs in practice, it is also indicative of the more dynamic nature of the Mean-CVaR approach, because in this context the higher rebalancing activity is in line with the strong improvements in downside risk measures and performance measures. (Novy et al., 2016).

Overall, table 6.2.1 shows that the MIM Mean-CVaR portfolio provides the best balance between return and risk among the strategies considered: although some alternative portfolios achieve similar returns, none combines return potential, volatility control, protection against extreme losses, and risk-adjusted performance as effectively, confirming that including sectoral information in the Mean-CVaR optimization process can improve portfolio construction by maintaining returns while reducing exposure to severe negative outcomes.

### 5.2.2 Minimum-Risk Comparison

Table 5.2.2 - Comparative Performance of Minimum-Risk Portfolio Strategies

| Strategy            | Minimum-CVaR SIM | Minimum-CVaR MIM | Minimum Variance | Equal Weight | S&P 500 |
|---------------------|------------------|------------------|------------------|--------------|---------|
| Mean monthly return | 0,54%            | 0,56%            | 0,59%            | 0,64%        | 0,60%   |
| Monthly variance    | 0,15%            | 0,14%            | 0,10%            | 0,34%        | 0,27%   |
| VaR95_Loss          | 6,64%            | 6,16%            | 4,90%            | 9,91%        | 8,46%   |
| CVaR95_Loss         | 8,81%            | 8,39%            | 8,13%            | 15,80%       | 13,66%  |
| Max Drawdown        | -30,77%          | -32,64%          | -33,64%          | -54,25%      | -51,37% |
| Sharpe Ratio        | 0,43             | 0,45             | 0,59             | 0,28         | 0,32    |
| Skewness            | -0,82            | -0,67            | -1,28            | -0,67        | -0,77   |
| Kurtosis            | 4,70             | 3,69             | 6,23             | 7,99         | 6,87    |
| Sortino Ratio       | 0,54             | 0,63             | 0,65             | 0,31         | 0,36    |
| Calmar Ratio        | 0,19             | 0,18             | 0,20             | 0,11         | 0,11    |
| Average Turnover    | 0,54             | 0,53             | 0,35             | /            | /       |
| Jensen Alpha        | 3,14%            | 3,34%            | 3,45%            | -0,25%       | /       |
| Annual return       | 5,70%            | 5,96%            | 6,58%            | 5,71%        | 5,66%   |
| Annual volatility   | 13,33%           | 13,10%           | 11,20%           | 20,21%       | 17,91%  |

Source: author's elaboration using MATLAB outputs.

The current section concentrates on portfolios designed primarily to reduce risk: table 5.2.2 presents the realized performance of the Minimum-CVaR portfolios obtained from the SIM and MIM approaches alongside that of the Minimum Variance portfolio, the Equal Weight allocation, and the S&P 500 benchmark, because all three optimization methods explicitly emphasize risk containment, this comparison provides valuable insight into how different definitions of risk influence portfolio construction and realized outcomes; a key finding concerns the overall return pattern: although the portfolios are primarily designed to reduce risk, all minimum-risk allocations generate annual returns between approximately 5.7% and 6.6%, a narrow range that indicates that lower risk does not necessarily lead to a substantial reduction in performance. The Maximum annual return is recorded for the Minimum Variance portfolio (6.58%), followed by the Minimum-CVaR MIM portfolio (5.96%) and the Minimum-CVaR SIM portfolio (5.70%); nevertheless, consistent with the previous section, return metrics alone offer only a partial assessment of portfolio quality and should therefore be evaluated together with the corresponding risk measures.

Regarding overall return variability, the Minimum Variance portfolio yields the smallest monthly variance (0.001) and the lowest annual volatility (11.20%), performing better than both Minimum-CVaR portfolios, whose annual volatility remains around 13%, this result aligns with the aim of the Minimum Variance approach, which is designed to minimize variance directly; by contrast, the Minimum-CVaR strategies concentrate on restricting losses in the worst part of

the return distribution, therefore, a marginally higher total volatility should not be viewed as evidence of less effective risk management, this is reflected in the downside-risk results: although the Minimum Variance portfolio delivers the lowest VaR loss (4.90%) and a marginally lower CVaR loss (8.13%), the difference relative to the Minimum-CVaR portfolios is relatively small; specifically, the MIM portfolio records a VaR of 6.16% and a CVaR of 8.39%, while the SIM variant achieves 6.64% for VaR and 8.81% for CVaR, therefore, during the sample period, the Minimum Variance strategy records relatively low losses in the left tail and the MIM Minimum-CVaR portfolio shows a similar level of robustness, despite being based on a different risk measure.

Another important indicator is maximum drawdown, defined as the greatest cumulative loss recorded from a prior portfolio peak: the Minimum-CVaR portfolios outperform the Minimum Variance approach in terms of drawdown behaviour, in particular, the Minimum-CVaR SIM portfolio shows the lowest maximum drawdown across the set (−30.77%), whereas the MIM variant has a value of −32.64%, in contrast, the Minimum Variance portfolio exhibits a more pronounced maximum drawdown of −33.64%; these outcomes highlight an important difference between variance-based optimization and CVaR-based optimization: lower average variability does not necessarily provide better protection during prolonged periods of market stress (Rockafellar & Uryasev, 2000). When considering the Sharpe Ratio the Minimum Variance portfolio records the highest Sharpe value (0.59), reflecting its combination of relatively strong returns and low volatility: the Minimum-CVaR MIM portfolio follows with a Sharpe Ratio of 0.45, while the SIM portfolio achieves 0.43; consistent evidence is also provided by the Sortino Ratio and the Calmar Ratio, for which the Minimum Variance approach attains the highest values, therefore, within the minimum-risk set, the variance-based portfolio is especially successful in translating realized risk into superior returns (Sharpe, 1964; Sortino & van der Meer, 1991).

Focusing on the shape of Minimum-CVaR portfolio return distributions, it is interesting to notice that both strategies show less negative skewness than the

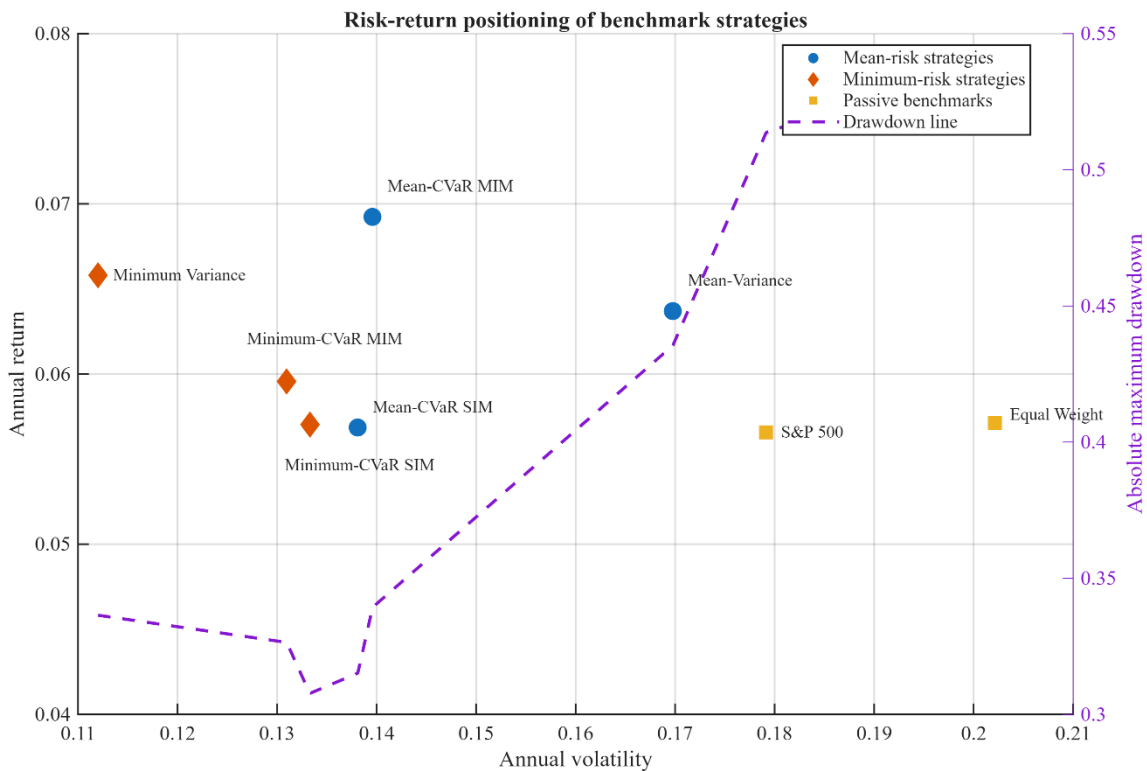
Minimum Variance strategy: the MIM portfolio has a skewness of  $-0.67$ , compared with  $-1.28$  for the Minimum Variance portfolio, indicating a less pronounced asymmetry toward extreme negative returns; likewise, the kurtosis values for the Minimum-CVaR portfolios are lower, pointing to a diminished likelihood of extreme tail realizations, from a risk-management standpoint, these distributional properties are crucial, as they suggest a more even dispersion of outcomes rather than a mere reduction in volatility (Rockafellar & Uryasev, 2000). The Jensen's alpha results further support the assessment of portfolio performance: in all three optimized specifications, the estimated alpha is positive, showing that realized returns exceed those attributable solely to exposure to market risk, the largest alpha is obtained for the Minimum Variance portfolio ( $0.0345$ ), which is closely followed by the Minimum-CVaR MIM portfolio ( $0.0334$ ) and the SIM version ( $0.0314$ ); the observed differences are modest, indicating that each optimized portfolio adds value relative to a passive market strategy (Jensen, 1968). A further aspect to consider is turnover: the Minimum Variance produces a lower average turnover ( $0.35$ ) than the Minimum-CVaR portfolios, for which turnover is closer to  $0.54$ , indicating that variance-driven optimization results in relatively more stable portfolio weights over time, whereas the Minimum-CVaR strategy updates allocations more frequently as the portfolios' downside characteristics change (Novy et al., 2016).

These results show that the Minimum Variance portfolio performs best when risk is measured using traditional volatility indicators, achieving the highest annual return, the lowest volatility, and the strongest risk-return performance; despite these results, the Minimum-CVaR portfolios remain competitive, particularly when the analysis focuses on downside protection and the nature of portfolio losses, in this regard, the MIM Minimum-CVaR approach generates smaller drawdowns, less pronounced negative skewness, and VaR and CVaR values that are similar to those of the Minimum Variance benchmark.

### 5.2.3 Discussion of Results

The comparison in the preceding section shows that portfolio performance should not be assessed solely on the basis of realized returns: each portfolio construction method pursues different objectives, thus its effectiveness must be evaluated by considering both the returns generated and the level of risk taken to achieve them; comparing the portfolios with the benchmark therefore provides a more complete assessment of the practical implications of the optimization methods examined in this thesis.

*Figure 5.2.1 - Risk–Return Positioning of Benchmark Portfolio Strategies*



*Source: author's elaboration using MATLAB outputs.*

Figure 5.2.1 summarizes the outcomes by showing all portfolio strategies according to their risk and return: the horizontal axis reports annualized volatility, whereas the vertical axis shows annualized returns, then each point captures the combined effect of realized performance and the associated level of risk; moreover, the dashed line represents the absolute value of the maximum drawdown, reported along the secondary axis, thereby enabling an integrated

assessment that accounts for both return variability and the extent of the most severe cumulative loss observed over the considered period. The figure highlights the main trade-off between the different allocation strategies: the passive benchmarks (the Equal Weight portfolio and the S&P 500 index) appear on the right side of the chart, indicating relatively high volatility without a corresponding return advantage; in particular, the Equal Weight portfolio shows the highest volatility, only moderate performance, and the largest drawdown among all strategies, suggesting that simple diversification alone does not necessarily produce an efficient risk–return balance (De Miguel et al., 2009).

The optimized portfolios cluster in a clearly distinct portion of the graph: both Minimum-CVaR and the Minimum Variance portfolios are concentrated in the lower-volatility region, consistent with their defensive orientation: the Minimum Variance strategy delivers the lowest observed risk together with the highest return, highlighting the empirical merits of variance defensive allocations reported in the literature; at the same time, the Minimum-CVaR portfolios continue to perform well, offering a risk characterization that is explicitly tied to adverse outcomes (Clarke et al., 2011).

The most important results concern the mean-risk approaches: the Mean–Variance portfolio generates a relatively high annual return, but this result is accompanied by considerably greater volatility; by contrast, the Mean–CVaR portfolios, especially the MIM formulation, achieve returns that are comparable to, or even higher than, those of the Mean–Variance benchmark while maintaining a lower level of risk, as a result, these strategies occupy a more favourable position in the risk–return space, indicating that they transform the risk assumed into realized returns more efficiently (Pun et al., 2023). This finding is particularly relevant because it captures the main reason for adopting the Mean–CVaR strategy which is designed to produce more stable investment outcomes by explicitly limiting exposure to extreme losses, and the comparison with the benchmark confirms that this objective is effectively achieved; the Mean–CVaR portfolios consistently produce lower downside risk, smaller drawdowns, and better performance relative

to risk without compromising return generation, providing a more balanced overall outcome (Rockafellar & Uryasev, 2000).

The figure further confirms the advantages of incorporating sector information through the Multi-Index Model: under both the mean-risk and minimum-risk configurations, the portfolios constructed using the MIM consistently lie above those based on the SIM, implying higher realized returns at equivalent risk levels; this is in line with the results reported in the previous chapter and indicates that a more informative characterization of the dependence structure among assets can lead to more effective portfolio choices (Tulloch et al., 2020).

In conclusion, the results show that the Mean–CVaR optimization can improve on conventional portfolio optimization methods because explicitly accounting for extreme losses produces portfolios with stronger downside protection and greater resilience in adverse market conditions; the integration of CVaR therefore leads to more robust portfolio decisions, particularly when combined with the more detailed dependence structure provided by the Multi-Index Model.

## Conclusion

The empirical analysis presented in this thesis demonstrates the importance of accurately modelling the dependence structure between asset returns when generating scenarios for portfolio optimization; the Single-Index Model captures shared movements in securities' returns by modelling their exposure to the market factor but the analysis of the residuals shows that a significant degree of dependence remains, especially among stocks operating in the same sector.

The Multi-Index Model, by incorporating sector information captures part of this common variation not explained by the market factor and provides a more complete representation of the relationships between securities; this improvement has important implications for portfolio construction: when the scenarios produced by the two models are used in the same Mean–CVaR optimization process, portfolios based on the MIM achieve better out-of-sample performance than those based on the SIM, generating higher returns and stronger risk-adjusted results without a significant increase in overall volatility; however, this advantage does not apply to all risk measures, as the MIM portfolios experience a slightly higher maximum drawdown, but in general the inclusion of the sector component leads to portfolio allocations that perform more effectively throughout the evaluation period.

The comparison with the other benchmarks supports these results: in the mean-risk analysis, the Mean–CVaR approach achieves a better balance between returns and downside protection than the traditional Mean–Variance model and among all the strategies considered, the MIM specification produces the strongest overall performance; the minimum-risk analysis leads to similar conclusions: portfolios based on Minimum CVaR offer greater protection against adverse outcomes than traditional Minimum Variance portfolios because they explicitly account for extreme losses.

Collectively, these outcomes provide a clear answer to the main research question: by considering both market and sector effects, the Multi-Index Model offers a more informative basis for generating return scenarios and combined with Mean–CVaR

optimization, it also leads to better out-of-sample results and stronger performance relative to the level of risk taken; these findings underline the importance of including sector information in the investment process and support the MIM–Mean–CVaR approach as an effective method for constructing portfolios in uncertain market conditions.

## Bibliography

- Aabo, T., Pantzalis, C., & Park, J. C. (2017). Idiosyncratic volatility: An indicator of noise trading?. *Journal of Banking & Finance*, 75, 136-151.
- Alexiou, C., & Tyagi, A. (2020). Gauging the effectiveness of sector rotation strategies: evidence from the USA and Europe: C. Alexiou, A. Tyagi. *Journal of Asset Management*, 21(3), 239-260.
- Anand, V., & Ramasubramanian, S. V. (2015). *Generalized Herfindahl-Hirschman Index to Estimate Diversity Score of a Portfolio across Multiple Correlated Sectors* (pp. 1-7). IFF Working Paper Series.
- Bailey, D. H., & de Prado, M. L. (2021). How “backtest overfitting” in finance leads to false discoveries. *Significance*, 18(6), 22-25.
- Baker, S. R., Bloom, N., Davis, S. J., Kost, K., Sammon, M., & Viratyosin, T. (2020). The unprecedented stock market reaction to COVID-19. *The review of asset pricing studies*, 10(4), 742-758.
- Bessler, W., Taushanov, G., & Wolff, D. (2021). Factor investing and asset allocation strategies: a comparison of factor versus sector optimization. *Journal of Asset Management*, 22(6), 488-506.
- Bhojraj, S., Lee, C. M., & Oler, D. K. (2003). What's my line? A comparison of industry classification schemes for capital market research. *Journal of accounting research*, 41(5), 745-774.
- Blume, M. E., & Stambaugh, R. F. On the Assessment of Risk. *77je Journal of Finance* (March 1971), 1-10.
- Campbell, J. Y., Lo, A. W., MacKinlay, A. C., & Whitelaw, R. F. (1998). The econometrics of financial markets. *Macroeconomic Dynamics*, 2(4), 559-562.
- Cashin, P., Mohaddes, K., & Raissi, M. (2017). China's slowdown and global financial market volatility: is world growth losing out?. *Emerging Markets Review*, 31, 164-175.
- Chan, L. K., Lakonishok, J., & Swaminathan, B. (2007). Industry classifications and return comovement. *Financial Analysts Journal*, 63(6), 56-70.
- Chekhlov, A., Uryasev, S., & Zabarankin, M. (2005). Drawdown measure in portfolio

- optimization. *International Journal of Theoretical and Applied Finance*, 8(01), 13-58.
- Clarke, R., De Silva, H., & Thorley, S. (2011). Minimum-variance portfolio composition. *Journal of Portfolio Management*, 37(2), 31.
- DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?. *The review of Financial studies*, 22(5), 1915-1953.
- Echterling, F., & Eierle, B. (2015). Mean reversion adjusted betas used in business valuation practice: A research note. *Journal of Business Economics*, 85(7), 759-792.
- Elton, E. J., Gruber, M. J., Brown, S. J., & Goetzmann, W. N. (2014). *Modern portfolio theory and investment analysis* (9th ed.). Wiley, pp. 126-173.
- Fairbrother, J., Turner, A., & Wallace, S. W. (2022). Problem-driven scenario generation: an analytical approach for stochastic programs with tail risk measure: J. Fairbrother et al. *Mathematical Programming*, 191(1), 141-182.
- Ferreira, F. G., & Cardoso, R. T. (2021). Mean-CVaR Portfolio Optimization Approaches with Variable Cardinality Constraint and Rebalancing Process: FGDC Ferreira, RTN Cardoso. *Archives of Computational Methods in Engineering*, 28(5), 3703-3720.
- Fulkerson, J. A., & Riley, T. B. (2019). Portfolio concentration and mutual fund performance. *Journal of Empirical Finance*, 51, 1-16.
- Grégoire, V. (2020). The rise of passive investing and index-linked comovement. *The North American journal of economics and finance*, 51, 101059.
- Han, R. Q., Xie, W. J., Xiong, X., Zhang, W., & Zhou, W. X. (2017). Market correlation structure changes around the great crash: A random matrix theory analysis of the Chinese stock market. *Fluctuation and Noise Letters*, 16(02), 1750018.
- Herskovic, B., Kelly, B., Lustig, H., & Van Nieuwerburgh, S. (2016). The common factor in idiosyncratic volatility: Quantitative asset pricing implications. *Journal of Financial Economics*, 119(2), 249-283.
- Hollstein, F., Prokopczuk, M., & Simen, C. W. (2020). Beta uncertainty. *Journal of*

*Banking & Finance*, 116, 105834.

Horenstein, A. R. (2021). The unintended impact of academic research on asset returns:

The capital asset pricing model alpha. *Management science*, 67(6), 3655-3673.

James, N., Menzies, M., & Gottwald, G. A. (2022). On financial market correlation structures and diversification benefits across and within equity sectors. *Physica A: Statistical Mechanics and its Applications*, 604, 127682.

Jensen, M. C. (1968). The performance of mutual funds in the period 1945-1964. *The Journal of finance*, 23(2), 389-416.

Jiang, B., Liu, C., & Tang, C. Y. (2024). Dynamic covariance matrix estimation and portfolio analysis with high-frequency data. *Journal of Financial Econometrics*, 22(2), 461-491.

Keating, C., & Shadwick, W. F. (2002). A universal performance measure. *Journal of performance measurement*, 6(3), 59-84.

Kim, D. (2015). Beta vs. characteristics: Comparison of risk model performances. *Journal of Empirical Finance*, 34, 156-171.

Liu, Y., Mauer, D. C., & Zhang, Y. (2018). The hedging benefits of industrial and global diversification: Evidence from economic downturns. *Journal of Business Finance & Accounting*, 45(9-10), 1322-1351.

Luan, F., Zhang, W., & Liu, Y. (2022). Robust international portfolio optimization with worst-case mean-CVaR. *European Journal of Operational Research*, 303(2), 877-890.

Manda, K. (2010, April). Stock market volatility during the 2008 financial crisis. In *Seminar study presented to the Leonard N. Stern School of Business*.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, 7(1), 77-91.

Markowitz, H. M. (1959). Portfolio selection: Efficient diversification of investments. John Wiley & Sons, 5-7.

Mazur, M., Dang, M., & Vega, M. (2021). COVID-19 and the march 2020 stock market crash. Evidence from S&P1500. *Finance research letters*, 38, 101690.

Michaud, R. O., & Michaud, R. (2007). Estimation error and portfolio optimization: a resampling solution, 5(4), 1-25.

- MSCI Inc., & S&P Dow Jones Indices. (2024). *Global Industry Classification Standard (GICS®) Methodology*. MSCI Inc.
- Novy-Marx, R., & Velikov, M. (2016). A taxonomy of anomalies and their trading costs. *The Review of Financial Studies*, 29(1), 104-147.
- Ponomareva, K., Roman, D., & Date, P. (2015). An algorithm for moment-matching scenario generation with application to financial portfolio optimisation. *European Journal of Operational Research*, 240(3), 678-687.
- Pun, C. S., Wang, T., & Yan, Z. (2023). Data-driven distributionally robust CVaR portfolio optimization under a regime-switching ambiguity set. *Manufacturing & Service Operations Management*, 25(5), 1779-1795.
- Rockafellar, R. T., & Uryasev, S. (2000). Optimization of conditional value-at-risk. *Journal of risk*, 2, 21-42.
- Securities and Futures Commission. (2014). *A review of the global and local securities markets in 2013*. Research Paper No. 54, 1–18.
- Sharpe, W. F. (1963). A simplified model for portfolio analysis. *Management science*, 9(2), 277-293.
- Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, 19(3), 425-442.
- Simaan, M., Simaan, Y., & Tang, Y. (2018). Estimation error in mean returns and the mean-variance efficient frontier. *International Review of Economics & Finance*, 56, 109-124.
- Sortino, F. A., & Van Der Meer, R. (1991). Downside risk. *Journal of portfolio Management*, 17(4), 27.
- Staino, A., & Russo, E. (2020). Nested Conditional Value-at-Risk portfolio selection: A model with temporal dependence driven by market-index volatility. *European Journal of Operational Research*, 280(2), 741-753.
- Strub, M. S., Li, D., Cui, X., & Gao, J. (2019). Discrete-time mean-CVaR portfolio selection and time-consistency induced term structure of the CVaR. *Journal of Economic Dynamics and Control*, 108, 103751.
- Tulloch, D. J., Diaz-Rainey, I., & Premachandra, I. M. (2020). Modelling sector-level

asset prices. *Journal of Risk and Financial Management*, 13(6), 120.

Wang, Y., Pan, Z., Wu, C., & Wu, W. (2020). Industry equi-correlation: A powerful predictor of stock returns. *Journal of Empirical Finance*, 59, 1-24.

Yan, C., & Zhang, H. (2017). Mean-variance versus naïve diversification: The role of mispricing. *Journal of International Financial Markets, Institutions and Money*, 48, 61-81.

Zurek, M., & Heinrich, L. (2021). Bottom-up versus top-down factor investing: an alpha forecasting perspective: M. Zurek, L. Heinrich. *Journal of Asset Management*, 22(1), 11-29.