Department of Economics and Management

Master’s degree in finance

Beta Forecast Accuracy and Mean-Variance Portfolio Performance: A Comparison of Raw, Blume, and Vasicek Methods

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Abstract

In both practical and empirical capital market research, the beta factor is typically estimated for future periods using OLS regression on historical data. However, these betas are neither reliable for forecasting dynamic portfolio performance nor for achieving statistical accuracy. Therefore, methods have been developed to improve the stability of beta estimates and to adjust historical beta estimates. The aim of this study is to examine the extent to which forecast accuracy and portfolio performance, based on raw beta, can be improved by using the Blume and Vasicek beta. For this purpose, an empirical analysis is conducted based on the full sample of stocks in the S&P 500 index. Over the entire sample period, it can be observed that although both Vasicek and Blume adjustment methods tend to improve statistical accuracy compared to raw beta, adjustment methods do not improve portfolio performance. Analysis of the COVID-19 pandemic period shows that statistical forecast accuracy is lower for all methods compared to the entire sample period, and that adjustment methods still perform better than raw beta. During the initial phase of COVID-19 period, portfolio cumulative returns and Sharpe ratio decreased for all methods relative to the overall period, with the Vasicek adjustment outperforming raw beta. However, after the initial stage of the COVID-19 period, adjustment methods are unable to perform better than raw beta. Overall, while beta adjustment methods improve statistical accuracy, adjustment methods do not improve portfolio performance relative to raw beta.

Sommario

In entrambe le ricerche pratiche ed empiriche sui mercati dei capitali, il fattore beta è tipicamente stimato per periodi futuri utilizzando la regressione OLS su dati storici. Tuttavia, questi beta non sono né affidabili per prevedere la performance dinamica del portafoglio né per raggiungere un'accuratezza statistica. Pertanto, sono stati sviluppati metodi per migliorare la stabilità delle stime di beta e per aggiustare le stime di beta storiche. L'obiettivo di questo studio è esaminare la misura in cui l'accuratezza delle previsioni e la performance del portafoglio, basate sul beta grezzo, possono essere migliorate utilizzando il beta di Blume e di Vasicek. A tal fine, viene condotta un'analisi empirica basata sull'intero campione di azioni dell'indice S&P 500. Su tutto il periodo campione, si può osservare che, sebbene entrambi i metodi di aggiustamento Vasicek e Blume tendano a migliorare l'accuratezza statistica rispetto al beta grezzo, i metodi di aggiustamento non migliorano la performance del portafoglio. Analisi del periodo della pandemia di COVID-19 indica che l'accuratezza delle previsioni statistiche è inferiore per tutti i metodi rispetto all'intero periodo campione, sebbene i metodi di aggiustamento continuino a performare meglio del beta grezzo. Durante la fase iniziale del periodo COVID-19, i rendimenti cumulativi del portafoglio e il rapporto di Sharpe sono diminuiti per tutti i metodi rispetto al periodo complessivo, con l'aggiustamento Vasicek che ha sovraperformato il beta grezzo. Tuttavia, dopo la fase iniziale del periodo COVID-19, i metodi di aggiustamento non riescono a performare meglio del beta grezzo. Nel complesso, mentre i metodi di aggiustamento del beta migliorano l'accuratezza statistica, i metodi di aggiustamento non migliorano la performance del portafoglio rispetto al beta grezzo.

Chapter 1 Introduction

1.1 What is Beta and Why is It Important?

Beta (β) is a measure of the systematic risk or volatility of individual stocks or a portfolio in comparison to the market. A beta of 1 indicates that stock returns are expected to move the same as market returns. A beta > 1 indicates that stock returns are expected to be more volatile than market returns. A beta < 1 indicates that stock returns are expected to be less volatile than market returns. In practical applications, beta is estimated from past beta and used as an estimate of future beta.

In both empirical capital market research and practical applications, beta is estimated using standard OLS regression, assuming beta remains constant over future periods, and without adjustment. In this case, the Blume and Vasicek adjustment methods are used to improve the predictive stability of historical beta. The CAPM assumes beta as a constant over the estimation period during which the regression of stock returns is performed on the market returns. However, empirical studies show that beta estimates obtained from OLS regressions are not constant over time. Because the risk of stocks changes over the estimation period, the value of beta varies over time, meaning that raw beta is not a reliable model for forecasting the risk of stocks in future periods. For this reason, the Blume and Vasicek methods were developed to adjust or correct the raw beta and thereby, improve forecasting the risk of stocks over time.

One possible approach to measure the risk of stocks by considering systematic risk, that is, the risk that affects the entire market, so that it is considered as inevitable risk, since it cannot be eliminated even through diversification. Recessions and wars are common examples of sources of systematic risk. Although this type of risk has an effect on the entire securities, unsystematic risk affects a specific type of industry or individual security. Therefore, this type of risk can be avoided through diversification. A portfolio with diversified securities can help investors avoid unsystematic risk. Beta is therefore an important input variable that enables investors and portfolio managers to measure the level of systematic risk, which cannot be avoided. It is important for investors to calculate an accurate forecasted beta in order to form portfolios that achieve the optimal risk-return trade-off and reduce industry- or security-specific risk. This can be accomplished by constructing a mean-variance portfolio that achieves the optimal risk-return relationship. However, a limitation of beta is that it is calculated from the historical returns of stocks and does not incorporate forward-looking adjustments. For example, raw beta does not account for future changes in a firm’s strategy or leverage. Consequently, raw beta is not a reliable measure for forecasting the risk of stocks in future periods.

This paper focuses on a comparison of beta models in the form of the raw, Blume, and Vasicek models by constructing a mean-variance portfolio and regression analysis. The paper is structured as follows: Chapter 2 discusses the literature review by including the theoretical framework of adjustment models and portfolio optimization, then discussing the paper's findings for comparison between raw betas and adjusted betas. Chapter 3 studies procedures, and in Chapter 4, the results of the procedures are presented. In the last section, some final remarks and potential ideas for future studies will be presented.

Chapter 2 Literature Review

2.1 Theoretical Framework of Raw Beta, Blume Beta, Vasicek Beta and Mean-Variance Portfolio

2.1.1 Raw Beta

Calculation of historical beta estimation is simply done by regression of each stock return on market returns. This can be indicated as

$$\mathrm{R_i} = \alpha_i + \beta_i \mathrm{R_m} + \epsilon_\mathrm{i} \quad (1)$$

Although the values of alpha; α_i, beta of stocks; β_i, error term; ϵ_i (which indicates deviation of observed values from their true values in the model) and the variance of error term $\sigma^2\epsilon_i$ changes over time, the equation (1) remains constant only if these parameters assumed to constant through the estimation period. Parameters: α_i, β_i and $\sigma^2\epsilon_i$ cannot be observed using historical data; instead, only historical returns of stocks and the market can be observed. However, if α_i, β_i and $\sigma^2\epsilon_i$ are assumed constant then with same equation, these parameters can be estimated by simply running the regression of the stock returns on the market returns over fixed historical period.

The equation (1) describes a linear regression model that can be represented as a straight line. If the variance of the error term, which captures the parts of the returns not explained by the model, is zero, the model perfectly explains stock returns with no deviation from actual values. With only two pairs ($\mathrm{R_i}$, $\mathrm{R_m}$) model can estimate β_i and α_i. When the random error term exists, actuals stock returns will differ from the expected return, which is predicted by using one factor model.

This results in data points being spread around the regression line, as shown in figure (1), which plots stock returns against the market returns. The slope of straight line is the beta of stock. This slope shows how stock returns are volatile against the market returns over the full sample period from February 2005 to February 2025. The x-axis represents returns on the S&P 500, whereas the y-axis illustrates returns of a stock randomly chosen from the U.S. equity market (represented by the S&P 500 index). Each point represents a particular period of stock return that plotted against the market return for same period. The actual observations lie on the solid line or near the solid line. However, if the value $\sigma^2 \epsilon_i$ is large then points will deviate more from the solid line. Therefore, significant part of the risk of the stock returns cannot be explained by the movement in the market. This indicates that systematic risk is not only factor that affects risk of stock returns, but firm-specific risk is also an important factor that affects risk of stock returns, which cannot be captured only by the market returns.

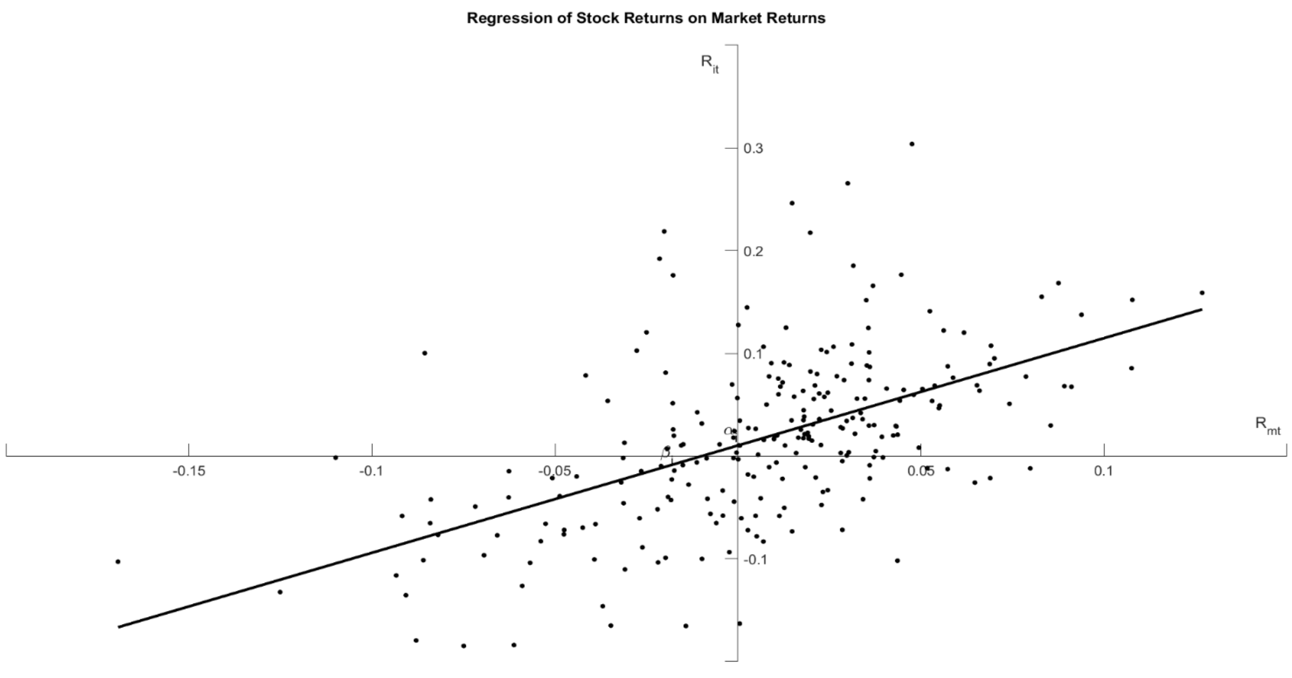

Figure 1: regression of randomly selected stock returns from S&P 500 equity against market returns

We first plot the historical returns of a stock against market returns to obtain a scatter plot, as shown in Figure 1, where each point represents the stock return and the market return at a particular point in time. In this case, monthly returns are used, and the scatter plot is constructed by plotting stock returns against market (S&P 500) returns over a 20-year period. The next step involves fitting a straight line to the data that minimizes the sum of squared deviations from the line in the vertical (R_{it}) direction. The slope of the fitted line represents the estimate of beta, together with the intercept, over the total period. More formally, raw beta can be calculated by dividing the covariance between the security return and the market return by the variance of the market return. Calculation of beta for each stock via regression analysis is shown as:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{\sum_t (R_{it} - \bar{R}_i)(R_{mt} - \bar{R}_m)}{\sum_t (R_{mt} - \bar{R}_m)^2} \quad (2)$$

Where $\beta_i =$ beta for individual stock i; ${\sigma^2}_m$ = Variance of market return and $\sigma_{im} =$ Covariance between security i and market return. $\bar{R}_i$ = average return of stock i and $\bar{R}_m$ = average return of market. Intercept estimation done with $\alpha_i = \bar{R}_i - \beta_i \bar{R}_m$

2.1.2 Blume Technique

Is it possible to improve the accuracy of historical beta estimation? To answer this question, let's address two main reasons why we need to use two adjustment techniques to improve the accuracy of beta estimation. First is the relation to the distribution of beta. If we estimate beta for all securities, some securities beta will be 1, some of them will be lower than 1, and some of them will have a beta higher than 1 this can be due to sampling error in the estimation of beta. Those securities with beta above 1 are simply due to positive sampling error whereas securities with a beta lower than 1 are due to negative sampling error. If this argument is correct, then on the average, betas tend to converge 1 in the following periods; thus, predicting beta by the standard OLS method did a worse job than predicting beta by the adjustment methods.

The second reason relates to fundamental factors in a firm's financial performance. Firms with low beta may subsequently take on more debt to finance additional projects, meaning that firms operating with low leverage in one period can shift to higher leverage in the next, which increases their risk profile. As a result, the beta estimated in the previous period will not remain constant in the following period, and it is possible for beta to increase and move toward one. Blume suggested that although firms that take on substantial debt may exhibit betas above one and be more volatile than the market, their betas will, over time, converge to the market beta (systematic risk). Blume's technique aims to capture this tendency by adjusting historical betas toward 1, under the assumption that the adjustment observed in previous periods provides a reasonable estimate for the next period.

2.1.2 Vasicek Technique

Vasicek showed that even if firms risk profile remains constant or does not exposure to market risk, mean reversion will still occur due to statistical reason, which can be considered as sampling error under estimating beta by standard OLS. In contrast to Blume, Vasicek technique does not perform same adjustment process for all stocks, rather each stock's beta adjusted according to their statistical quality of OLS estimation. This approach ensures that for each type of stock, specific adjustment process is considered so that adjustment process does not perform same amount mean- reversion for all stocks in the sample. Thus, instead of shrinking every stock's beta by the same amount, each stock's beta is shrunk toward its own prior mean. The extent of the adjustment depends on the size of the sampling error of the OLS beta: the larger the sampling error, the higher the variance of the OLS estimator, and the greater the adjustment applied under the Vasicek model. The adjustment is therefore larger for securities with higher beta standard errors than for those with lower standard errors.

Vasicek (1973) proposed the following scheme that incorporates these characteristics: if $\bar{\beta}_1$ equal average beta across stocks in the sample then Vasicek procedure takes a weighted average of $\bar{\beta}_1$and historical OLS beta for securities. Let $\sigma^2_{\bar{\beta}_{i1}}$to denote the variance of betas across stocks in the sample. This is a measure of the variation of beta across stocks in the sample. Let $\sigma^2(\beta_{i1})$ represent the square of the standard error of the OLS beta for stock i. This measures uncertainty in the OLS beta estimation for the stocks that Vasicek (1973) proposed weights of $\frac{\sigma^2_{\bar{\beta}_1}}{\sigma^2_{\bar{\beta}_1}+\sigma^2_{\beta i1}}$ for β_{i_1} and $\frac{\sigma^2_{\beta_{i1}}}{\sigma^2_{\bar{\beta}_1}+\sigma^2_{\beta i1}}$ for $\bar{\beta}_1$. These weights are sum up to 1, and the more uncertainty in OLS beta estimation, the lower weight placed on it. The forecast of beta for security *i* in the next period can be shown by

$$\beta_{2i} = \frac{\sigma^2_{\beta i1}}{\sigma^2_{\bar{\beta}_1} + \sigma^2_{\beta i1}}\bar{\beta}_1 + \frac{\sigma^2_{\bar{\beta}_1}}{\sigma^2_{\bar{\beta}_1} + \sigma^2_{\beta i1}}\beta_{i_1} \quad (3)$$

This equation shows adjusted beta is the weighted average of the stocks historical beta and the cross-sectional mean beta. If the standard error of β_i is higher compared to the cross-sectional variance of all betas ($\sigma^2_{\bar{\beta}_1}$) in the sample, the historical beta of security is considered less reliable. Therefore, greater weight is assigned to cross-sectional mean, and less to the stock beta, so that the adjusted beta for stock is shrunk more toward the cross-sectional mean. In the opposite case, when the estimate of stock beta is considered reliable (low standard error), this will lead to a higher weight of the stock beta than cross-sectional mean of beta, resulting in less shrinkage. Moreover, the cross-sectional variance ($\sigma^2_{\bar{\beta}_1}$) is decreased by shrinkage of individual beta estimates toward cross-sectional mean of betas. This adjustment procedure also applies to Blume procedure (Staal and Flint, 2025). Based on the equation, a similarity between Blume and Vasicek can be shown:

$$\beta^{Vasicek}_{i,2} = (1 - \lambda)\,\bar{\beta}_1 + \lambda\beta_{i,1} \quad (4)$$

$$\lambda = \frac{\sigma^2_{\bar{\beta}_1}}{\sigma^2_{\bar{\beta}_1}+\sigma^2_{\beta i1}},\ 1\text{-}\ \lambda = 1 - \frac{\sigma^2_{\bar{\beta}_1}}{\sigma^2_{\bar{\beta}_1}+\sigma^2_{\beta i1}} = \frac{\sigma^2_{\bar{\beta}_1}+\sigma^2_{\beta i1}}{\sigma^2_{\bar{\beta}_1}+\sigma^2_{\beta i1}} - \frac{\sigma^2_{\bar{\beta}_1}}{\sigma^2_{\bar{\beta}_1}+\sigma^2_{\beta i1}} = \frac{\sigma^2_{\beta i1}}{\sigma^2_{\bar{\beta}_1}+\sigma^2_{\beta i1}}$$

Similarity arises because both methods decrease cross-sectional variance through shrinkage, while the Blume adjustment is based on cross-sectional regression and applies fixed coefficients to all stocks in the sample, the Vasicek estimator uses a statistical shrinkage approach in which weights are different for each stock in the sample.

One advantage of the Vasicek over the Blume technique is that beta can be adjusted not only against the market but also against a specific sector average or stock sample. According to Gray (2008), the difference between the Vasicek and Blume adjustments comes from purely statistical factor; having observed a low beta or high beta in the corresponding period, a statistical adjustment is applied to correct for the statistical bias caused by estimation error. Notice that under this rationale, true beta does not revert toward one over time at all, but the adjustment is performed as a statistical correction only. Vasicek correction is clearly a statistical adjustment to correct bias because of estimation error in the OLS beta estimation. Since it is based on Bayes' Rule, this correction does not relate to the mean reversion of true betas over time.

2.1.3 Mean Variance Optimization

This section first discusses the main idea of the mean-variance portfolio with and without a riskless asset, and then focuses on how different choices of beta affect the mean-variance construction within a one-factor model (the CAPM framework).

Consider a market with a n number of securities which have same expected returns and variances where expected returns denoted by E [Ri] = μ and variances denoted by Var (Ri) = σ^2 for any i = 1, 2…n. Also, we assume that covariances between expected returns for i and j securities are zero. Let w_i represents fraction of wealth invested in the securities and total amount of wealth invested in these securities must sum to the 1

$$\sum_{i=1}^{n} w_i = 1$$

and let W_0 denotes the initial wealth that is available for the investment then amount invested in each asset can be written as

$$W_i = w_i W_0$$

Therefore, total portfolio return given by sum of weighted individual asset returns can be shown by $R_p = \sum_{i=1}^{n} w_i R_i$

Considering only two portfolios where portfolio A and portfolio B and suppose that investors invested all their wealth in portfolio A so that total amount fraction of wealth invested in portfolio A is $w_1=1$ and $w_i=0$ for all i and if we assume second portfolio B is invested by weighted equally which can be shown by $w_i=1/n$ for i=1,..n. Then expected return for portfolio A and expected return for portfolio B will be same with different variances where for portfolio A variance denoted by $Var(R_A) = \sigma^2$ and for portfolio B $Var(R_B) = \sigma^2/n$.

According to these given expected returns and variances any rational investors will immediately choose portfolio B because with portfolio B, investors benefit from diversification with same expected return. This is the core idea of the Markowitz framework, in which investors seek to minimize variance for a given expected return or maximize expected return for a given level of risk. In a portfolio consisting of two or more random variables, the relationship between asset returns is measured by their covariance. Let R_1 denote return for first asset and R_2 denote return for second asset and with expected values $E[R_1]$ and $E[R_2]$ the covariance between these two assets can be computed by

$$\mathrm{Cov}(R_1, R_2) = E[(R_1 - E[R_1])(R_2 - E[R_2])] \quad (5)$$

The covariance between two assets is often denoted as σ_{R_1,R_2} or by symmetry σ_{R_2,R_1} also there is more shortened version for computing covariance between random two random variables can be shown by

$$\mathrm{Cov}(R_1, R_2) = E[R_1 R_2] - E[R_1]E[R_2] \quad (6)$$

According to the mean-variance criterion under the Markowitz framework, a portfolio is considered efficient if no other portfolio allows investors to achieve a higher expected return at the same level of risk, or a lower level of risk at the same expected return. Three concepts are fundamental. The first is the opportunity set (or feasible region), which includes all efficient and inefficient portfolios available to investors.

The second is the minimum-variance frontier, which contains only portfolios with the lowest variance for any given target level of expected return. This frontier considers only risk allocation, regardless of expected returns.

The third is the efficient frontier, which includes only those portfolios that offer the highest expected return for a given level of risk. This region lies above the global minimum-variance portfolio (GMVP): above the GMVP, investors can target a higher expected return for the same level of risk. These concepts form the foundation of the Markowitz portfolio theory and are central to investors' decision-making.

The discussion can also be made for efficient frontier with riskless free asset. Let A denote risky portfolio that lies on the efficient frontier it has expected return μ_A and risk (standard deviation) σ_A , we need to include risky portfolio with risk-free asset let x is the fraction invested in the risky portfolio A and 1-x is the fraction invested in the riskless asset. Portfolio of expected return with riskless rate given by

$$\mu_p = x\mu_A + (1 - x)R_f \quad (7)$$

portfolio return is the weighted average of risk and riskless return, and formula of standard deviation can be interpreted by $\sigma_p = x\sigma_A$, risk-free asset not included in the standard deviation because risk-free asset has zero variance solving x and substituting into expected return

$$\mu_p = R_f + \frac{\mu_A - R_f}{\sigma_A}\sigma_p \quad (8)$$

equation of straight line obtained with intercept R_f and slope is

$$\frac{\mu_A - R_f}{\sigma_A} \quad (9)$$

This is equation called capital market line. The formula $\frac{\mu_A - R_f}{\sigma_A}$ is called Sharpe ratio and shows the highest excess expected return for acquiring certain amount of risk, where risk is measured by standard deviation. A higher Sharpe ratio indicates a better risk-return trade-off, as the portfolio delivers a higher excess return relative to its risk. Under the assumptions of homogenous beliefs and frictionless markets, there exists a portfolio located at the point where the Capital Market Line is tangent to the efficient frontier, and this portfolio maximizes expected return for a given level of risk. Assuming there are no market frictions or taxes and that investors share homogenous beliefs, with all investors able to lend and borrow at the same risk-free rate, then all rational investors will hold the same tangency portfolio, as they will seek the maximum Sharpe ratio, that is, the portfolio that maximizes return for a given level of risk. Although investors differ in their risk preferences, they will all hold the same risky portfolio (the tangency portfolio) and simply combine it with the risk-free asset in different proportions. Highly risk-averse investors will hold mostly risk-free asset, with their optimal point moving down the frontier, whereas more risk-tolerant investors will hold a larger proportion of the tangency portfolio, with their optimal point moving up the frontier.

The following discussion will be focused on how to construct mean-variance portfolio based on how different choices of beta can impact the portfolio optimization problem in one factor model which is CAPM framework. Instead of using historical returns, within CAPM framework expected returns will be constructed from equation (3) by using betas as an input factor.

$$E(R_i) = R_f + \beta_i\big(E(R_m) - R_f\big) \quad (10)$$

Where Rf denotes the risk-free rate, E(Rm) represents the expected market return and β_i measures systematic risk of stock i. after computing expected return, mean-variance optimization for Markowitz portfolio constructed by

$$\min_{w} \frac{1}{2} w' \Sigma w \quad (11)$$

$$\text{subject to} \quad w' \mu = p$$

$$and \quad w' 1 = 1$$

where Σ is the covariance matrix asset returns, μ is the expected return vector from CAPM model and p is the portfolio target return. This quadradic optimization problem will be solved with Lagrange Multiplier methods and solving that optimization problem achieves the optimal portfolio weights. With different beta choices, expected return vector changes thus it alters the optimization constraints and leads to different optimal weights. If beta increases for asset, then from CAPM expected return of asset will be increases leads to asset be more desirable in the optimization problem because it contributes more toward for achieving target return and larger weight will be assigned to that asset, therefore optimal weight for allocation that asset will be changed. If risk-free security is assumed to be available with equal to r_f. Let $w := (w_1, \dots, w_n)'$ be vector for the portfolio weights that are assigned on risky assets, thus $1 - \sum_{i=1}^{n} w_i$; is the weight assigned on risk-free asset. Mean-variance Optimization problem can be constructed as

$$\min_{w} \frac{1}{2} w' \Sigma w \quad (12)$$

$$subject\ to \quad \left(1 - \sum_{i=1}^{n} w_i\right) r_f + w' \mu = p$$

This optimization will result in a set of efficient portfolios and among all efficient portfolios, investors will choose portfolio that maximizes Sharper ratio

$$\max_{w} \frac{w'(\mu - r_f 1)}{\sqrt{w' \Sigma w}} \quad (13)$$

With maximizing Sharper ratio, investors will obtain optimal weights for tangency portfolio as

$$w = \xi\Sigma^{-1}(\mu - r_f 1) \quad (14)$$

Since expected vector return μ computed through CAPM model, different beta estimates will result in different expected returns, therefore the constraint will be change and set of portfolios that satisfy target return of the portfolio changes. With CAPM formula

$$\mu - r_f = (E(R_m) - r_f)\beta \quad (15)$$

Changes in beta estimation directly changes the Sharper ratio thus optimal portfolio weights for Tangency portfolio also changes.

According to Best and Grauer (1991), mean-variance portfolio optimization is highly sensitive to changes in the expected return. They discuss that even small changes in expected return can lead to substantial changes in mean-variance optimization thus optimal weights. To analyze effect of changes in expected return directly for optimal weights, the author included explicitly in computation of mean-variance optimization with no risk-free asset that is subject to linear constraints as

$$\max\{ t\mu' x - \frac{1}{2} x'\Sigma x : Ax \le b\} \quad (16)$$

Where μ and x are vectors for expected return and portfolio weights, t is the investors risk-tolerance. Mean-variance optimization problem can be solved with fixed risk tolerance parameter as a T for investors to understand how optimal solution of equation (17) is depends on the μ. The equation formulated as

$$\max_x T(\mu + tq)^T x - x^T \Sigma x \quad (17) \quad \text{s.t. } Ax \le b$$

In this equation, T is considered as a fixed risk tolerance parameter, and a parametric quadratic programming approach is used, where the expected return vector varies as $\mu(t) = \mu + tq = \mu + \Delta\mu$. The term tq captures directional changes in expected returns. Value of q assigned as a β,the vector of betas from the CAPM model and this shows that expected return changes in proportion to betas. When $q = \beta$, direction of change for expected return depends on beta and the size of the change for expected return controlled by the t. Optimal mean-variance portfolio weights can be formulated through linear function of perturbation parameter, $x(t) = h_0 + th_1$, where optimal portfolio weights composed as constant portion of the optimal weights from the solution of the mean-variance optimization (h_0) and changes in the portion of the optimal weights from the solution of the mean-variance optimization (th_1) when expected return changes as the betas changes.

Therefore, proportion of changes in beta estimation for each asset can lead to changes in the expected return and even small changes in the expected return, particularly when assets have similar expected returns lead to altering the optimal portfolio weights. However, author also concluded that even portfolio weights change a lot, portfolio expected return changes slightly. Portfolio expected return formulated by

$$E[R_p] = x^T \mu \quad (18)$$

Change in the expected returns from one single factor formula (CAPM) leads to

$$dE[R_p] = x^\top d\mu + \mu^\top dx \quad (19)$$

Change in expected return of portfolio decomposed as direct effect change which denoted as $x^\top d\mu$ where expected return of individual asset changes while weights stay same and indirect effect change as $\mu^\top dx$ where expected return of assets did not change but only weights are changed so both changes offset each other therefore, expected portfolio return slightly affected from changes in the expected return even weights are changed. When expected return of asset increases, weight for that specific asset in the portfolio will be increased through re-allocating more weights to that specific asset by decreasing weights of the other assets which leads to offset portfolio of the expected return.

The idea of how changes in expected return changes optimal weights but artificially changes portfolio performance can be shown by using different beta choices thus changing expected return from CAPM model and illustrating efficient frontier and tangency portfolio with graph.

These different beta choices come from raw, fixed Blume coefficient and Vasicek with fixed period from January 2020 to January 2025 with using 5 stocks from SP500 to illustrate graph. To isolate effect of different beta choices risk-free rate and mean(market) is kept the same for 4 beta models. Figure (2) represents the efficient frontiers and capital market lines which are derived from different beta estimation methods.

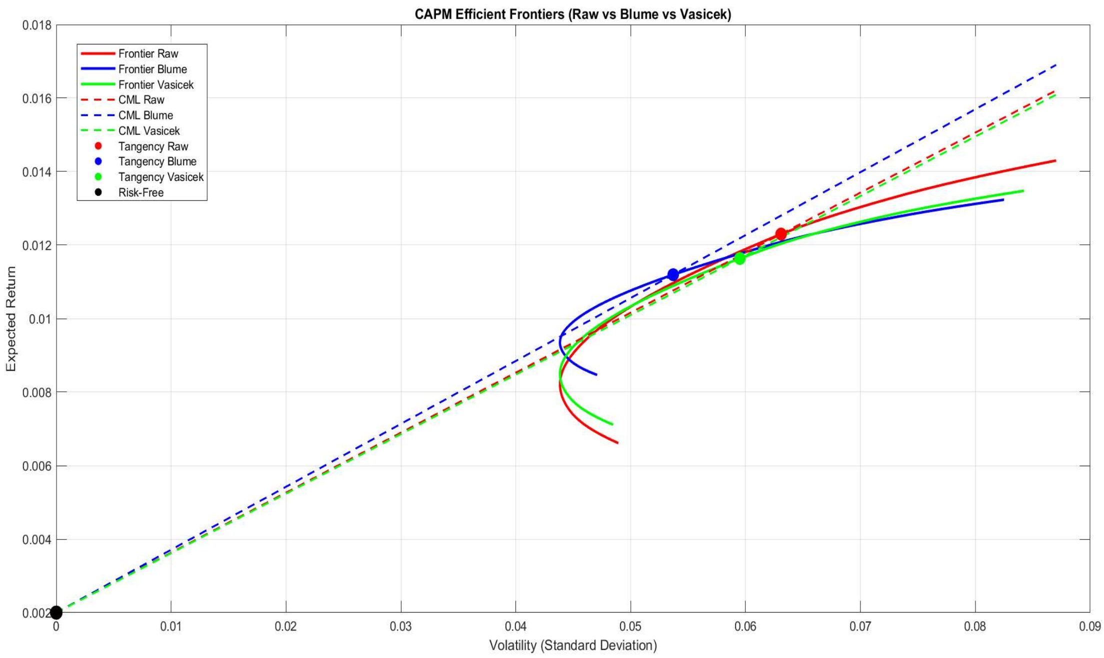

Figure 2: shows the CAPM efficient frontiers (Raw, Blume and Vasicek)

From figure (2) efficient frontiers are closely aligned with different beta models, which indicates for investors that optimal investment portfolios which offer highest expected return for given risk level are largely not affected by different beta estimation choices. Figure (2) also shows capital market line which represents optimal risk-return relationship for portfolios that combine risk-free asset with tangency portfolio.

Comparing the three Capital Market Lines, they exhibit similar slopes, suggesting only minor differences in the ex-ante Sharpe ratios, since the slope of the CML is the Sharpe ratio. The dots representing the tangency portfolio under each beta estimation model indicate that the maximum Sharpe ratio is similar across beta models. The tangency portfolio under the Blume and Vasicek estimation methods appears to lie slightly below that of raw beta due to shrinkage of the beta estimates in the CAPM model. This results in less dispersed expected returns and lower volatility. Next, it is possible to analyze how optimal tangency weights are changed for 5 randomly selected stocks under different beta estimation techniques.

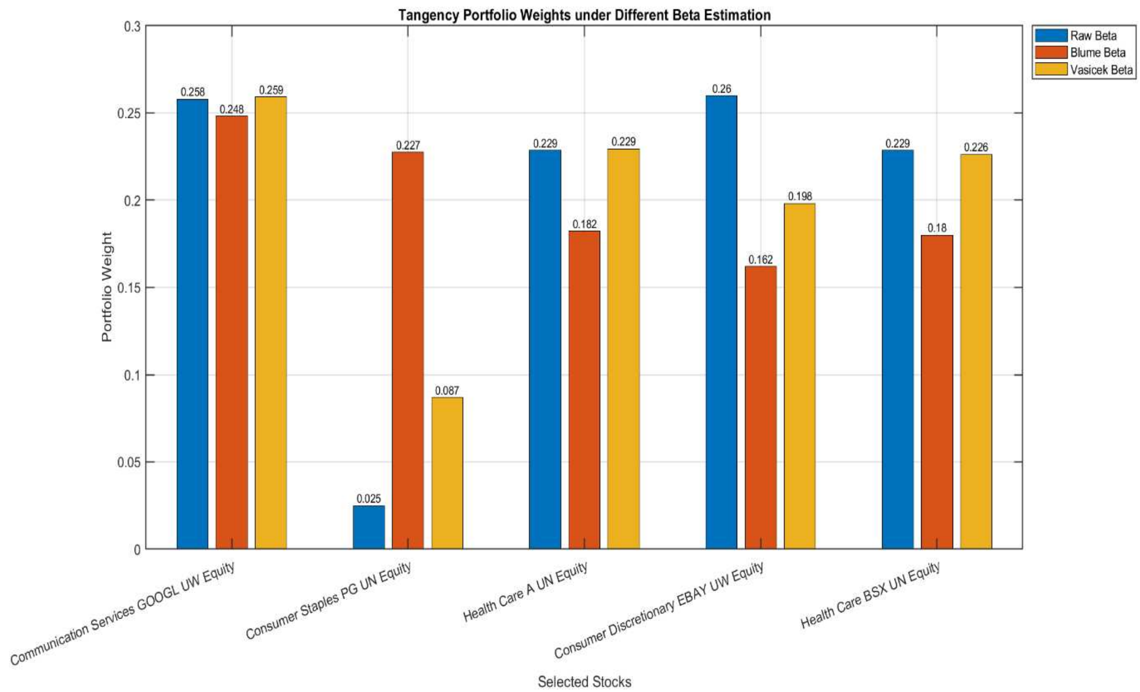

Figure 3: Tangency portfolio weights under different beta choices

Figure (3) shows how the tangency portfolio weights change when estimation of beta changes for the calculation of expected return from the CAPM model. While some assets exhibit significant changes in the optimal weight allocation across beta estimation methods, these differences are mainly between raw and Blume beta. Overall, both figures indicate that while optimal portfolio weights are highly sensitive when expected return changes due to different beta choices, tangency portfolio exhibits only minor differences in the optimal portfolio performance.

However, for illustrative purposes to show efficient frontiers and tangency portfolio, models are absent from rolling window, only limited number of stocks are used from SP500 index and fixed coefficients are used for Blume beta estimation which these coefficients can be vary across time and across different stocks so in the methodology part more advanced techniques will be used for evaluating portfolio performance under the raw, Blume and Vasicek estimation models.

2.2 Prior studies that evaluated beta dynamics and adjustment methods

2.2.1 Conditional and Dynamic Beta

Early investigations into how macroeconomic conditions, such as the business cycle, affect beta are provided by Jagannathan and Wang (1996). The authors introduced a conditional CAPM framework that examines how betas respond to changes in the market conditions. Their results showed that the assumption of constant beta in the unconditional CAPM is unrealistic, as time-varying betas were statistically significant because of the changes in the market conditions. This evidence supports the view that beta responds to changes in the market environment. Also, under the conditional CAPM, alpha value became close to zero and it is statistically insignificant which yields the model better to explain return with time varying of beta, so they concluded constant beta is not good model for forecasting beta because market conditions vary and beta need to be time varying.

Additionally, Bali, Cakici and Tang (2009) documented that market conditions affect beta estimates, supporting that it is advisable to condition beta models on current market environment. They found that conditional beta, which varies with economic conditions, provides a better explanation of cross section of expected returns than static beta and the relationship is positive and statistically significant.

These Studies reveal that assumption of constant beta is not appropriate during the extreme events for estimation beta of stocks. Studies highlight the fact that during extreme events, such as recession, business risk profile is different from that during expansion. Dynamic beta models are required to capture variations in beta across different phases of the business cycle. Unlike static beta model, which calculates risk of securities over fixed period, betas are needed to update during the estimation period for securities. Thus, the papers demonstrated that the relationship between market returns and individual stock returns is not constant as business life cycle evolves through time, supporting the need for time-varying beta models.

2.2.2 Adjusted Beta

Blume (1975) observed that betas tend to move toward 1. He corrected past betas by adjusting the betas toward 1 for forecasting betas of stocks for next periods. The regression trend in betas can be estimated using cross-sectional data on stocks betas over two consecutive periods. Betas of stocks in the cross-sectional sample in period t, as $\beta_{i,t}$ regressed on the previous period t-1 as $\beta_{i,t-1}$, the formula can be shown as:

$$\beta_{i,t} = a_t + b_t \beta_{i,t-1} + \varepsilon_{i,t} \;\; (20) \;\; for\; i = 1,2,\dots,n$$

Where n represents the number of stocks in the cross-sectional sample, a and b represent coefficients in period t, and $\varepsilon_{i,t}$ represent a random error term. To account for autoregressive behavior of beta when forecasting beta, adjustment methods can be applied to consider the dependence of stock's current beta on its previous period. The betas of the stocks in the cross-sectional sample are regressed for two consecutive periods to obtain Blume regression coefficients by using cross-sectional regression. The estimated Blume coefficients are then used to forecast beta for the next period by the following formula:

$$\beta_{i,t+1} = a_t + b_t \,.\, \beta_{i,t} \; (21)$$

He practically estimated betas through 1948-1954 also he estimated beta for 1955-1961 then he regressed same stocks for the period 1955-1961 on the betas for the period 1948-1951. Blume coefficients are estimated by using cross-sectional regressions of stock betas from (1948-1951) and (1955-1961), where betas in later period are regressed on betas in the earlier period, across all stocks in the sample. From that regression the following coefficients are obtained

$$\beta_{i,1955-1961} = \frac{1}{3} + \frac{2}{3}\,\beta_{i,1948-1954}$$

$\beta_{i,1955-1961}$ represents beta of stocks in the later periods and $\beta_{i,1948-1954}$ represent beta of stocks in the earlier periods. Regression formula and these coefficients demonstrate that betas of stocks in later periods $\frac{1}{3} + \frac{2}{3}$ times beta in earlier periods. With this procedure Blume obtained that forecasted beta for later periods is closer to the 1 than beta estimated through historical beta. After obtaining the Blume regression coefficients from equation (18), the betas for the period 1962-1968 were forecasted by applying the Blume adjustment formula to the historical beta estimates, which is calculated for the period 1955-1961. Forecast of beta for next period can be shown by

$$\beta_{\widehat{1962-1968}} = 0.343 + 0.677\,\beta_{1955-1961}$$

This adjusted formula lowers the values of high beta whereas raises the lower values of betas in the earlier periods when forecasting betas of stocks for later periods. Suppose that the historical beta ($\beta_{i,1}$) is 2.0 then the beta for next period is $\beta_{i,2} = 0.343 + 0.677 \times 2 = 1.697$. Similarly, if the historical beta is 0.5 then the beta for next period is $\beta_{i,2} = 0.343 + 0.677 \times 0.5 = 0.682$. Once the Blume coefficients are obtained, for each stock historical beta is multiplied by the slope coefficient and adding the intercept provides the Blume forecasted beta for next periods that is closer to 1 than the raw historical forecasted beta.

According to Klemkosky and Martin (1975), they tested ability of these models to forecast over each five years periods for both individual stock and 10 portfolio stock, they found that, in all cases, Blume and Vasicek techniques led to more accurate forecast of future beta than estimating through unadjusted beta model in terms of MSE. The average squared error in forecasting beta was lower in both adjustment methods than raw betas, and error in Vasicek method slightly lower than Blume method. Klemkosky and Martin (1975) state that forecast error arises partly from misestimating the average level of beta, and partly from the tendency to underestimate low betas and overestimate high betas. When the Vasicek and Blume adjusted methods are compared to the unadjusted method, most of the reduction in forecast error results from a decline in this tendency. Klemkosky and Martin found that the Vasicek model outperforms the Blume model in terms of forecasting accuracy, although the difference between them is small.

Dimson and Marsh (1983) examined the stability of beta estimates for thinly traded securities after applying a method that corrects for thin-trading bias. The results showed that individual security betas exhibited only moderate stability, whereas portfolio betas were stable. Furthermore, when security betas were adjusted using the Blume and Vasicek techniques, the coefficients on the adjusted betas indicated improved forecast accuracy. However, Bera and Kannan (1986) tested the data and found deviations from normality. They therefore concluded that the Blume and Vasicek methods are not ideal, since both assume that estimation errors are normally distributed. If the errors have heavy tails, the adjustments may not perform as intended.

According to Gray et al. (2014), the Australian Energy Regulator (AER, 2009) has not implemented adjusted betas for Australian-listed stocks. This is because benchmark firms do not alter their long-term debt and equity structure or diversify over time; accordingly, the true beta is not expected to converge toward one, and no adjustment models are applied. Gray et al.

agree that the mean-reversion rationale is unlikely to apply to benchmark firms. This motivates their use of the Vasicek correction rather than the Blume method, since their primary aim was to reduce statistical bias caused by estimation error, based on Bayes' Rule.

Gray et al. (2014) evaluate Vasicek and OLS method by sorting firms into low-, medium-, and high-beta portfolios due to larger variations in the market cap of securities in the sample. They compute expected returns for each portfolio using the CAPM, with β_i from Vasicek and OLS method, and regress realized portfolio returns on CAPM expected return. From CAPM- based analysis they concluded that Vasicek betas are better to explain realized returns than OLS betas in terms of R squared. However, difference in term of the R squared only 0.001 and when paper tested CAPM with raw and Vasicek beta, both failed to accept the intercept term as zero means that estimated intercept was statistically different from zero but paper accepted Beta as one means that estimated Beta was not statistically different from one under the raw and Vasicek beta.

Sinha and Jayaraman (2012) calculated MSFE for Blume adjustment, classical method and Vasicek adjustment and they observed that Vasicek method produced lower MSFE, indicating that betas were closer to future values than classical model. Moreover, they noticed that Blume technique captures under and over estimation of betas by using two consecutive time periods to find the slope coefficient (b) in the Blume beta adjustment regression, and researchers found b is less than one so this leads to high betas to shrink downward and low betas to rise upward toward the mean. Therefore, this corrects over and under estimation in beta measure. However, even with this information Vasicek method and classical method still outperformed Blume method due to lower value in MSFE for Vasicek beta.

Mokta Rani Sarker (2013) done empirical study on forecasting ability of different estimation methods to estimate beta and hypothesis testing done to find out is there any difference between to estimate future betas on the context of Bangladesh during the specific periods. His study was conducted on listed of individual stocks rather than stock portfolio. Hypothesis tests done to evaluate performance of Blume, Vasicek and raw betas by analyzing ANOVA table. From ANOVA table by calculating F values, he observed that Blume and Vasicek forecasted beta is significantly different than actual beta. Overall conclusion of paper is that neither Blume's nor Vasicek's adjustment techniques provide accurate forecasts of future betas for the Bangladeshi stock market during the sample period due to inefficient scenario of Bangladesh stock market.

Kian Shan (2025) investigated accuracy of forecasting stock beta by computing raw beta, Blume and Vasicek technique and paper compared which methods provides better estimation of stocks beta. The analysis was conducted on daily returns for stocks whose market capitalization exceeded \$1 billion over the 17 years. Performance was evaluated by Diebold-Mariano test and standard regression metrics. Researcher calculated Blume adjusted beta by applying formula of $\beta_{\mathrm{adj}} = \beta_{\mathrm{raw}} \times \frac{3}{4} + \frac{1}{4}$. Then he investigated adjusted beta by using Bayesian approach by using different prior distributions. The first prior specification set based on cross-sectional distribution of OLS estimated betas and intercept. The others based on more restrictive prior on betas and intercepts so only first one is based on empirical evidence, which is observed from data while alternative priors are based on more restrictive assumptions about the distribution of beta and intercepts rather than empirical evidence from data. The larger differences between models arise from regression coefficients. Regression analysis showed that Blume method generates least biased predictor due to Blume method acquires lowest intercept. Then they examined coefficient of the beta term. This measures how much predicted beta value changes when changes occur in the actual beta. A slope of one illustrates perfect one to one relationship between predicted and actual value, however none of the Bayesian model and standard OLS estimation closer to one than Blume model over the entire sample period. This demonstrates that Blume model is better than Vasicek and raw beta for capturing true relationship between actual and predicted beta.

Second method is The Diebold Mariano test is a set of t-test to determine mean difference in forecast loss between two forecasts. From the t-test results, Vasicek and Blume method performed better than raw beta so relying solely on OLS estimate without any adjustments causes worst forecasting performance among all models. Another observation from the t-test was that Blume method performed better than Vasicek method. Overall, the results in the paper suggest that the Blume model provides best statistically reliable forecasts of future betas.

Chapter 3 Scope of Research

This study examines the performance of alternative beta estimation methods, including raw beta, Blume beta, and Vasicek beta. These betas are estimated by using a 36-month rolling window approach in order to capture time variation and reduce short-term estimation noise. The analysis evaluates the forecasting accuracy of these methods using statistical forecast performance metrics. In addition, dynamic mean–variance portfolios are constructed to assess the economic significance of each beta estimation technique in terms of risk–return performance. In addition to total period, which covers February 2005 to January 2025, Covid-19 period is also analyzed from March 2020 to February 2022. However, to better understand how dynamic portfolio performance metrics are sensitive to market shocks during covid period, an additional analysis conducted on the initial phase of the Pandemic for the portfolio evaluation.

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Chapter 4 Data and Methodology

4.1 Data

The performance of the Vasicek, Blume, and raw beta models is evaluated in terms of portfolio performance and forecast accuracy using S&P 500 stocks. The analysis covers a 20-year period and focuses on large-cap American stocks to minimize the dispersion in stock returns relative to the market. All stock prices are obtained from the Bloomberg database. To ensure a balanced sample over as long a period as possible, the analysis covers stocks from February 2005 to January 2025 and is based on the full sample of S&P 500 constituent stocks. Beta estimates are based on end-of-month closing prices, and monthly returns are calculated from these prices as the percentage change between consecutive months, as shown by

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad (22)$$

Descriptive statistics will be presented to provide an overview of the data. Summary statistics for stock returns, market returns and historical beta are reported both for full sample and Covid period. These statistics characterize distributional properties of data which includes the mean and standard deviation also skewness and kurtosis. Table 1 presents the descriptive statistics for the total period, which covers the 20 years from February 2005 to January 2025 of stock and market returns.

Table 1: **Descriptive Statistics for Full Period**

	Mean	Std	Min	Max	Skewness	Kurtosis
Stock Returns	0.0152	0.0711	-0.2935	0.3212	0.0351	4.0223
Market Returns	0.0074	0.0323	-0.0758	0.0924	-0.1243	3.8046
Historical Beta	1.0291	0.3945	0.2612	2.6265	0.18435	3.5179

From table 1 it can be concluded that, mean of stock returns and standard deviation is higher than market returns whereas kurtosis for stocks and market is larger than 3 which indicates that data is not normally distributed and data have fat tails. Negative skewness for market can be interpreted as when individual stocks are aggregated into market, systematic shocks affect all stocks simultaneously. As these events are typically associated with large negative returns, they generate a persistent left tail in the distribution, resulting in negative skewness at the market level. Moreover, for a more intuitive analysis, descriptive statistics focusing on the pandemic period are presented in Table 2

Table 2: Descriptive Statistics for Covid period (March 2020 to March 2022)

	Mean	Std	Min	Max	Skewness	Kurtosis
Stock Returns	0.0204	0.0956	-0.1532	0.2478	-0.0732	4.0971
Market Returns	0.0182	0.0544	-0.1251	0.1268	-0.5121	3.9538
Historical Beta	1.0341	0.4953	0.1323	2.9678	0.7054	4.7632

From table 2, during the Covid 19 period, standard deviation (volatility) increased for both stock and market returns also dispersion in the historical beta increased this can be because of during covid some stocks exhibit high beta and some stocks low beta which causes larger spread across stocks. Skewness of beta also increased this indicates that dataset have more extreme high-beta stocks and right tail becomes heavier than left tail.

Mean return of stock experienced high average returns during pandemic which is driven by the recovery periods however volatility also increased due to high uncertainty during Covid. Skewness of stock is still near to zero with slightly increased downside risk. Kurtosis is still higher than 3, suggesting extreme return realizations at the monthly frequency.

4.2 Methodology

For each month, at the end of February 2005 rolling raw beta is calculated based on past 36 month returns up to January 2025 and the study will be conducted for entire period and Covid period. For the evaluation of beta models, first method is to evaluate estimated beta models by constructing mean-variance portfolio. The classical mean–variance framework is inherently a static, single-period model. In practical portfolio management, however, asset returns and risk characteristics evolve over time. To account for this time variation, the static mean–variance optimization is implemented within a rolling-window framework. Each month, model parameters are re-estimated, and the portfolio is re-optimized accordingly. This approach preserves the theoretical foundation of the Markowitz model while allowing dynamic portfolio adjustments. This will be done using a 36-month rolling beta model in the CAPM formula to compute one-month-ahead expected returns

$$\mu_i = r_f + \beta_i(\mu_m - r_f) \quad (23)$$

μ_i is the expected return of asset i and μ_m is the expected return of market. Expected returns are derived from beta estimates obtained using realized returns, so that both beta estimation and expected return calculation are based on the same return series, ensuring that the forecasts are directly comparable with realized returns used in the evaluation of out-of-sample portfolio performance.

The Market return is represented by the monthly return of S&P 500, which is obtained directly from the Bloomberg dataset. In each month, expected market return is calculated using a 36-month rolling window as the arithmetic average of market returns from t-35 to t and it can be interpreted as

$$\widehat{\mu_{m,t}} = \frac{1}{36} \sum_{k=t-35}^{t} R_{m,k} \quad (24)$$

Where $R_{m,k}$ denotes the S&P 500 return in month k. Returns from February 2005 to January 2008 will be used to estimate raw beta, Blume beta and Vasicek beta. After that, forecast of expected returns will be made for the next month, which is February 2008, and this process will continue for subsequent months.

Since Blume beta needs cross-sectional regression for finding coefficients of Blume, Blume estimated beta will first be available in February 2008; thus, the forecast of expected return will be made in March 2008. Raw beta for January 2008 is estimated using monthly returns from February 2005 to January 2008 based on a 36-month rolling window and raw beta for February 2008 is estimated by using returns from March 2005 to February 2008.

To obtain the Blume adjustment coefficients for February 2008, a cross-sectional regression is conducted across all stocks, where the dependent variable is the raw beta in February 2008 and the independent variable is the raw beta in January 2008. The regression takes the form: $\widehat{\beta_{i,\text{feb2008}}^{\text{raw}}} = a_{\text{feb2008}} + b_{\text{feb2008}} \cdot \widehat{\beta_{i,\text{jan2008}}^{\text{raw}}} + u_i$, the estimated coefficients $a_{Feb2008}$and $b_{Feb2008}$ are then used to construct the Blume-adjusted forecasted beta for the first available month in the rolling period which is March; $\widehat{\beta^{\text{Blume}}_{i,\text{Mar2008}}} = a_{\text{Feb2008}} + b_{\text{Feb2008}} \widehat{\beta^{\text{raw}}_{i,\text{Feb2008}}}$ and this Blume-adjusted forecasted beta is used in the CAPM formula to obtain the expected return for March 2008, and the process continues for subsequent months. This cross-sectional regression is repeated each month throughout the sample period, allowing the Blume coefficients to vary over time.

To calculate Vasicek betas, the cross-sectional mean and variance of rolling 36-month raw betas are computed at each month-end using S&P 500 stocks then each stocks raw beta is shrunk toward cross-sectional mean by weight that depends on the estimation of the individual beta. The degree of shrinkage depends on the relative magnitude of the estimation error in the individual stocks beta. Shrinkage of individual beta estimates toward the cross-sectional mean beta applied in this methodology by the formula

$$\hat{\beta}_{i,t}^{\;Vasicek} = w_{it}\hat{\beta}_{i,t} + (1 - w_{it})\bar{\beta}_t \quad (25)$$

and weight is calculated through $w_{i,t} = \frac{\sigma^2_{\beta,t}}{\sigma^2_{\beta,t} + SE\left(\widehat{\beta_{i,t}}\right)^2}$ where $\hat{\beta}_{i,t}$ denotes the rolling 36-month raw beta for stock i, $\bar{\beta}_t$ represent cross-sectional mean and $\sigma^2_{\beta,t}$ represent cross sectional variance. Using a rolling window of 36 months and a forecast horizon of 1 month, the raw betas of all stocks are first estimated as of January 2008 using returns from February 2005 to January 2008.

Based on the betas of all stocks at the same time t, the cross-sectional mean and variance are computed in January 2008, and the Vasicek-adjusted beta for each stock is obtained at that date and expected return is computed in February 2008. However, to align with Blume beta all three models will be used to compute expected return from March 2008. The procedure is repeated at each subsequent month-end throughout the sample period.

Due to high dimensionality of datasets, cross-sectional regression and the rolling window approach, portfolio weights are allocated by maximizing mean-variance utility which is computationally more efficient and consistent with the Markowitz framework. After finding expected return for assets in each month investors portfolio optimization formulated with $\max_{w} \ \mu^{\top} w - \frac{1}{2} w^{\top} \Sigma w$ subject to subject to $1^{\top} w = 1, w \geq 0$, where short selling is not allowed. Unlimited assumption of short selling can be unrealistic. Investors can get extreme longs and extreme shorts thus extreme portfolios can be created. When we estimated covariances and returns then to construct portfolio by Markowitz we can get large positive weights (extreme longs) on some of the securities and large negative weights (extreme shorts) on some of the securities which can be unrealistic. When short selling is not allowed portfolio weights should be non-negative values.

This portfolio optimization is solved by quadprog to get optimal portfolio weights for each month. The expected portfolio return E(Rp) is calculated by combining expected returns of each individual security by taking account their optimal weights. The weight assigned to i-th asset in portfolio denoted as w_i and expected return vector denoted as μ

$$E(R_p) = w^T \mu = \sum_{i=1}^{n} w_i \mu_i \quad (26)$$

The variance of portfolio σ_P^2 is calculated by utilizing covariance matrix of security returns. w_j and w_i represent the weights assigned to security j and security i accordingly. The covariance is calculated from historical returns between security i and security j is covariance ($R_i R_j$).

$$\sigma_P^2 = \sum_{i,j} w_i w_j \ \mathrm{Cov}(R_i, R_j) \quad (27)$$

The Sharpe ratio of portfolio is used as an indicator for evaluating returns in relation to risk and it can be computed by expected return of portfolio, risk-free rate and standard deviation of portfolio.

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \quad (28)$$

The study will evaluate beta models for portfolio performance by computing out of sample Sharpe ratio to measure realistic performance so optimal weights are calculated at the time t based on the expected return under different beta choices then these weights are applied to realized returns where optimal weights are different across beta models but realized returns are same across all beta models at the time t+1 to find realized portfolio returns then compute out of sample Sharpe ratio. This can be explicitly shown by

$$R_{p,t+1} = w_t^{\top} R_{t+1} \quad (29)$$

Where the weights are from the expected return $\mathrm{E(R)} = u$ and applied to realized returns at the time t+1 which can be demonstrated as R_{t+1} and obtain realized portfolio return at the time t+1 to calculate realized Sharpe ratio

$$\mathrm{Sharpe}_{\mathrm{out\ of\ sample}} = \frac{\mathrm{R}_{\mathrm{p,out\ of\ sample}} - \mathrm{R_f}}{\sigma\left(\mathrm{R}_{\mathrm{p,out\ of\ sample}}\right)} \quad (30)$$

Second method is to evaluate beta models by forecasting accuracy metrics. Forecast accuracy is evaluated by comparing forecasted betas with realized betas. To avoid using overlapping data when comparing forecasted betas and realized betas, forecast of Raw, Blume and Vasicek beta will be made ahead of 36-month.

For example, raw beta estimated using returns from February 2005 to January 2008 is used as the forecasted beta for January 2011. This forecast is then compared with the realized beta in January 2011, where the realized beta is estimated using returns from February 2008 to January 2011. Similarly, raw beta estimated from March 2005 to April 2008, and naïve beta used a forecast value for April 2011 and compared with realized beta in April 2011. This procedure will be used for evaluating metrics in regression analysis and it will continue for subsequent months.

Calculation of Blume beta first Blume coefficients which are a_t and b_t determined by regressing beta of stocks for the period of t+36 months on the beta of stocks for the period of t. This can be shown by

$$\beta_{i,t+36} = a_t + b_t\beta_{i,t} + \varepsilon_{i,t} \quad (25)$$

These coefficients are only available from January 2011 on. This can be shown by $\beta_{i,January\ 2011} = a_t + b_t\beta_{i,January\ 2008} + \varepsilon_{i,t}$ where cross-section of betas of January 2008 estimated from February 2005 to January 2008 and cross-section of betas of January 2011 estimated from February 2008 to January 2011 therefore, once Blume coefficient is determined by regression cross-sectional across stocks we can forecast beta for the following 36-month by

$$\hat{\beta}_{i,t+36|t} = a_t + b_t\beta_{i,t} \quad (26)$$

which is Blume adjusted formula and specifically; $\hat{\beta}_{i,January\ 2014} = a_t + b_t\beta_{i,January\ 2011}$ this procedure is repeated monthly throughout the sample period.

With Vasicek adjusted beta, using a rolling window of 36 months and a forecast horizon of 36 months, the raw betas of all stocks are first estimated as of February 2008 using returns from January 2005 to February 2008. Based on betas of all stocks at the same time t, the cross-sectional mean and variance are computed in February 2008, and the Vasicek-adjusted beta for each stock is obtained at that date. This Vasicek beta is then interpreted as the forecasted beta for February 2011 and same procedure applied for the following months. Moreover, estimated beta models evaluated only when Blume forecasted beta is available thus first evaluation for beta models will be conducted in January 2014.

For the mean squared error (MSE) evaluation difference between forecasted beta and realized betas calculated when all forecasted beta exist at the same period. Deviations from realized betas are analyzed in the basis of time series regression which means regression of actual betas on the forecasted betas at each month. MSE is separated into 3 parts which shows bias, inefficiency and random disturbance.

$$\text{MSE} = \frac{1}{n}\sum_{t=1}^{n}\left(\beta_{t,f} - \beta_{t,r}\right)^2 \quad (27)$$

$$\text{MSE} = \left(\overline{\beta_f} - \overline{\beta_r}\right)^2 + (1-b)^2 \cdot \sigma_{\beta_f}^2 + (1-R^2) \cdot \sigma_{\beta_r}^2 \quad (28)$$

Where t is the number of periods, n is the number of observations and the sum of squared variance is divided by n and $\beta_{t,f}$ is the forecasted beta that computed beta of period t-36 that used as predictor of beta for t and $\beta_{t,r}$ is the actual beta in t, $\overline{\beta_f}$ and $\overline{\beta_r}$ are the means of forecasted and realized betas where b is the slope of regression of realized betas on the forecasted betas and $\sigma_{\beta_f}^2$, $\sigma_{\beta_r}^2$ are the sample variances of the forecasted and realized betas.

R^2 indicates the fraction of the variation in the dependent variable that is explained by the independent variable(s) in the regression model. Nonzero values of inefficiency and bias indicate that systematic forecast errors have existed. The random component refers to the unexplained variation between the predicted values and the actual observed values for the beta factor. In addition to MSE, MAE is also used further to analyze forecasting quality of beta models because MSE assigns greater weight to large forecast errors due to squared deviations between predicted and realized betas.

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|\beta_{t,\text{forecast}} - \beta_{t,\text{raw}}\right| \quad (29)$$

The Theil's Inequality Coefficient (TIC) is used for direct comparison when comparing raw betas and adjusted betas for forecasting betas

$$TIC = \frac{\text{RMSE}}{\text{RMSE}_{\text{naïve}}} \quad (30)$$

With a value for TIC lower than 1 quality of prediction for beta models e.g. Blume estimation is better than the naïve estimation in the form of raw beta. Therefore, TIC serves as a relative measure of forecast accuracy, where lower values indicate better predictive performance compared to raw beta.

Chapter 5: Results and the Discussion

In the following section, study will discuss how different beta estimation models can affect forecast accuracy in terms of the MSE between predicted and actual beta values. Additionally, for the MSE components. After that, study will discuss how different beta estimation models can affect portfolio performance. Two periods will be considered for the result and discussion part. First one is the whole period which is from February 2005 to January 2025 and second one is the subperiod which covers the pandemic. Results of the empirical analysis of total periods for the forecast accuracy of beta methods are shown in Table 3.

Table 3: MSE and Components for total periods

Model	MSE	Bias	Inefficiency	Noise
Raw	0.260805	0.000714	0.085837	0.174254
Vasicek	0.197055	0.000049	0.023716	0.173291
Blume	0.184712	0.000240	0.003814	0.180657

From Table 3, the mean values for bias, inefficiency, and random disturbance sum to the reported mean MSE, confirming the correctness of the decomposition. Raw beta (the naïve estimator) exhibits the highest average MSE, indicating the weakest statistical forecast accuracy among the models. This is an expected outcome, since raw beta relies solely on historical data without any shrinkage or mean reversion. Therefore, with raw beta forecasting, it becomes more sensitive to changes in sample data.

Vasicek follows the second lowest MSE in contrast to Blume beta which is leads to lowest MSE on average. Blume adjusted beta model leads to lowest MSE comparison with other two models, can be mostly attributed for specifically considering the mean-reversion of beta over time through cross-sectional regression. Therefore, it captures more directly how beta evolves over time.

The Vasicek adjustment, by contrast, focuses on reducing estimation noise in the OLS beta by shrinking estimates toward the cross-sectional mean, but does not directly model how beta evolves over time as the Blume method does. The Blume adjustment improves forecast accuracy by capturing the relationship between past and future beta through cross-sectional regression. Nevertheless, this noise reduction still yields more accurate forecasts and therefore lower prediction errors relative to raw beta.

Turning to the MSE components in Table 3, bias is relatively small for all beta models. Vasicek has the lowest bias, as its shrinkage toward the cross-sectional mean centers the forecasts around the mean. However, all models produce near-zero bias, indicating that the beta forecasts are not systematically biased, no model consistently overestimates or underestimates realized beta. The average difference between forecasted and realized beta is therefore close to zero across models, even though individual forecasts can be too high or too low at specific points in time or for specific stocks. Since bias reflects only the difference in average levels, the other MSE components must also be examined when evaluating forecast accuracy. The inefficiency component measures how well the forecasted betas co-move with realized betas. Among the three models, Blume has the lowest inefficiency, which can primarily be attributed to the fact that it learns the relationship between past and future beta through cross-sectional regression, thereby adjusting the slope coefficient and capturing the time-varying dynamics of beta. The high inefficiency of raw beta confirms that the naïve beta estimator has a poor ability to track beta dynamics relative to the adjusted models.

Noise component is the largest contributor to the MSE across all models and noise is very similar across all beta models. A large portion of beta variation remains still unpredictable, so changing beta estimation method does not reduce noise much. Noise component in the Mincer-Zarnowitz decomposition is defined as the variance of residuals, Noise: σ_u^2 where the residual term u given by

$$u = \beta_{real} - (a + b \cdot \beta_{forecast})$$

This component captures the unexplained variation in the realized beta that cannot be accounted by using different forecasted beta techniques. Although different beta estimation improves systematic forecast errors, magnitude of error is still high. Table 4 presents the values for RMSE, MAE and TIC.

Table 4: RMSE, MAE and TIC for Total Period

Model	**RMSE**	**MAE**	**TIC**
Raw Beta	0.510691	0.379704	1.00000
Vasicek Beta	0.443909	0.336684	0.8692
Blume Beta	0.429781	0.325549	0.8416

In addition to previous table, RMSE, MAE and TIC are also reported to provide a more comprehensive forecast accuracy. While MSE penalizes larger errors more heavily due to it squares errors, RMSE reveals forecast error in the original scale and MAE captures the average absolute deviation, which is less affected by extreme values. From both tables, MAE is higher than MSE because most beta errors are less than 1 and RMSE values are higher or equal to the MAE since RMSE square the differences before averaging, it gives more weight to larger errors, which results from both tables show that RMSE is higher than MAE.

The results provided in Table 4 show that Blume beta leads to best values in the average in terms of RMSE, MAE and TIC. Consistent reduction in RMSE, MAE values indicate that Blume and Vasicek adjustments improve forecasting accuracy relative to the raw beta.

This is also verified by Theil's inequality coefficients, since both adjustment models are less than 1, it demonstrates that both adjusted beta models lead to smaller forecasted errors than raw beta. Focus also can be made on regression analysis of beta forecast models.

Table 5 reports the values of the Mincer-Zarnowitz regression of realized betas on predicted betas across stock-time observations. The intercept measures systematic bias and slope measures degree of calibration on the forecast betas. The R squared measures the proportion of variance in the realized variance that is explained by the predicted betas.

Table 5: Intercept, Slope and R squared

Model	**Intercept**	**Slope**	**R Squared**
Raw Beta	0.5433	0.4406	0.2310
Vasicek Beta	0.3908	0.6019	0.2384
Blume Beta	0.2082	0.7780	0.2060

Results from table 5 indicate that all models exhibit slopes where below 1, which suggest that forecasted betas failed to capture dispersion in realized betas thus, do not fully capture variation in realized betas. Since intercept is not equal to zero and values are greater than zero, it indicates that there is presence of systematic bias, which suggests that predicted betas tend to underestimate realized betas on average after accounting for the slope of coefficient. It should be noted that, although bias in MSE component is close to zero, the regression intercept reveals the presence of systematic forecast error. This is because the intercept adjusts to compensate for deviation of the slope from the 1.

Among the different beta estimation methods, both adjustment methods have a higher slope and lower intercept than raw beta which suggests there is improvement in terms of the calibration of forecast and reduction in the systematic bias and highest improvement occurred in the Blume adjusted beta. While difference in the R squared is not larger between models, Vasicek performs better relative to other models int term of R squared but still exhibits limited explanatory power. In terms of total period, it can be specified that all two adjustment beta models presented in this paper show that forecast quality was improved compared to raw beta model and overall, Blume adjusted beta leads to best values.

Whether this demonstration also applies to other periods, which are characterized by the covid-pandemic crises will be analyzed. Subperiod is evaluated from March.28,2020 to March.28,2022. Thus, it includes the covid pandemic. The corresponding betas from the period March.28,2017 to March.28,2020 were used for respective forecasts. Details of result will be given in Table 6,7 and 8.

Table 6: MSE and Components for subperiod

Models	MSE	Bias	Inefficiency	Noise
Raw Beta	0.291056	0.003718	0.082402	0.204935
Vasicek Beta	0.234058	0.006012	0.018033	0.210013
Blume Beta	0.214302	0.003195	0.002359	0.208748

Firstly, mean values for bias, inefficiency and noise add up to the Mean values as stated. Compared to total period the average of MSE values in subperiod is higher than total period for both raw beta and two adjusted beta models.

Forecasting of the realized beta performs worse in subperiod compared to total periods among all three models. This is expected outcome because of structural break in the covid period. Structural break implies that relationship between market and stock returns changed suddenly which makes relationship between stock and market returns unstable and leads to poor performance among all beta models compared to total period.

Also, Mean value bias is significantly higher in subperiod while inefficiency and random noise are slightly changed among all models. This is mainly because Covid created a systematic shift which creates systematic over and under estimation between forecasted betas and realized betas thus leading to higher increase in the bias rather than inefficiency and random disturbance on the average. During pandemic, many stocks moved similarly due to systematic risk, so errors became more similar across observations on the average which created less dispersion in the sample.

However, one similarity compared to total period is that both adjusted betas are outperformed raw beta in terms of MSE and each MSE components. Blume beta provides beta values in both total and subperiod while raw beta is the worst in both periods. The next table, which is Table 7, presents RMSE, MAE, and TIC for evaluating the statistical forecasting accuracy of different beta estimates during the pandemic period as indicated by subperiod. These additional measures are included to capture different aspects of forecast error, as each metric emphasizes different properties of the deviations between predicted and realized betas.

Table 7: RMSE, MAE and TIC for subperiod

Models	**RMSE**	**MAE**	**TIC**
Raw Beta	0.539496	0.400043	1.000000
Vasicek Beta	0.483795	0.351708	0.8968
Blume Beta	0.462927	0.334188	0.8581

The values of RMSE, MAE and TIC shown in Table 7 are similar to the values that are shown in the total period. However, their magnitudes are higher than the total period. Blume and Vasicek produce the smaller average absolute prediction error than raw beta.

Blume and Vasicek also minimize the large forecast errors which show better robustness during the volatility period. TIC value, which is lower than 1, confirms that adjustment models provided the best outcome than raw beta and Blume beta leads to best performance among all models. Finally, table 8 will provide the interception, slope and R squared for the subperiod from March 2020 to March 2022

Table 8: Intercept, Slope and R squared for subperiod

Models	Intercept	Slope	R Squared
Raw Beta	0.6095	0.4422	0.1940
Vasicek Beta	0.4480	0.6180	0.2017
Blume Beta	0.2358	0.7685	0.1869

Values indicated in Table 8 show that intercept, slope and R squared changed and slightly decreased compared to total period. However, in terms of slope and intercept, adjustment beta provides better values than raw beta and Blume beta provides the best values while R squared still same across beta models. Overall, in both total and subperiod adjustment models improved the values compared to raw beta and Blume provides the best value for the forecasting performance of beta.

Second evaluation of different beta estimation methods will be conducted on dynamic portfolio performance in terms of assessing their out of sample Sharpe ratio and cumulative portfolio returns for both total and COVID-19 period from March 2020 to March 2022. In addition, a shorter window representing the initial market shock is reported, covering December 2019 to April 2020 to illustrate how portfolio performance is sensitive to changes in market dynamics. Table 9 provides the values of out of sample Sharpe ratio also portfolio mean return and portfolio standard deviation for three beta models for total period.

Table 9: Sharpe Ratio, Portfolio Mean Return and Standard Deviation

Models	**Out of Sample Sharpe Ratio**	**Portfolio Mean return**	**Portfolio Standard Deviation**
Raw Beta	0.2146	0.0184	0.0764
Vasicek Beta	0.1230	0.0096	0.0614
Blume Beta	0.2160	0.0183	0.0757

Firstly, we can verify result of Sharpe Ratio with portfolio mean return and portfolio standard deviation. Dividing portfolio mean-return by portfolio standard deviation gives the approximate results from table however for CAPM computation, interest rate was used by fixing 0.002 at a monthly and subtracting interest rate from portfolio mean return then dividing by portfolio standard deviation gives exact values for Sharpe Ratio that indicated in Table 9.

Regarding Sharpe Ratio for analyzing portfolio performance, it can be seen that Vasicek Beta performs worse in portfolio performance since Vasicek Beta has the lowest Sharpe Ratio. Also, it should be noted that Vasicek beta provides the lowest and portfolio standard deviation and the lowest portfolio mean return. Vasicek estimation method yields the reduction in the portfolio mean return higher than the reduction in the portfolio standard deviation which suggests that reduction in the return is proportionally larger than reduction in the risk. This is mainly due to Vasicek shrinkage decreases the dispersion of beta estimates which makes expected returns to become more similar across stocks and portfolio weights become less concentrated in high expected returns, so it weakens the portfolio performance since quadprog optimization allocate more weights on lower expected return of stocks and allocate less weight on higher expected returns of stocks which leads to reduction portfolio mean return more than reduction the volatility.

However, with raw beta estimation method, expected return will become more dispersed in the CAPM model with this optimization will allocate larger weights for the higher expected returns yields high return relative to risk so leads to higher Sharpe ratio.

Using Blume adjustment model for dynamic portfolio allocation requires monthly cross-sectional regression across stocks so it leads to less shrinkage than Vasicek while stabilizing beta and ensuring more reliable expected returns which gives better weights in term of risk-return relationship however comparing with raw beta Blume beta almost gives same Sharpe ratio as raw beta. Therefore, adjustment methods do not improve portfolio performance compared to raw beta for total period.

To further analyze dynamic portfolio performance, cumulative returns of portfolio are plotted over time for different beta estimation methods, Cumulative returns provide a comprehensive view of how portfolio values evolve over the sample period, capturing both short-term fluctuations and long-term performance differences across models. This allows for a clearer comparison of how each beta estimation method affects the growth of portfolio wealth over time. The resulting trajectories are presented in Figure (4).

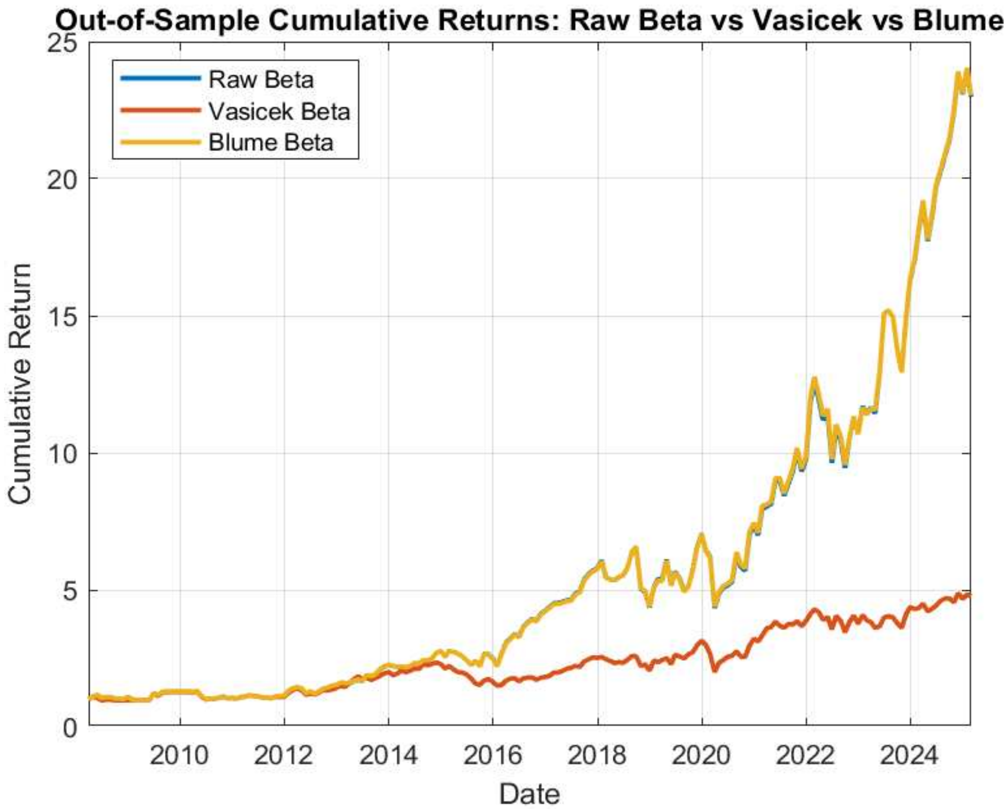

Figure 4 shows the out of sample cumulative returns under the raw, Blume and Vasicek beta

The figure (4) shows the total period of cumulative returns of portfolio which is constructed by using raw, Blume and Vasicek beta models. Cumulative returns look almost identical for Blume and raw beta whereas Vasicek model performs worse than both raw and Blume beta. However, it should be noted that from monthly cross-sectional regression across stocks, Blume coefficients obtained as, mean Blume slope is 0.9840 and intercept is 0.0160 for total period so this indicates that current betas are strong predictor of future betas and intercept implies weak tendency toward mean reversion. Consequently, constructing portfolio with Blume adjusted betas yields the almost same values as the raw betas over the full sample period. To understand dynamic evaluation of Blume coefficients, figure (5) illustrates the Blume slope for total period.

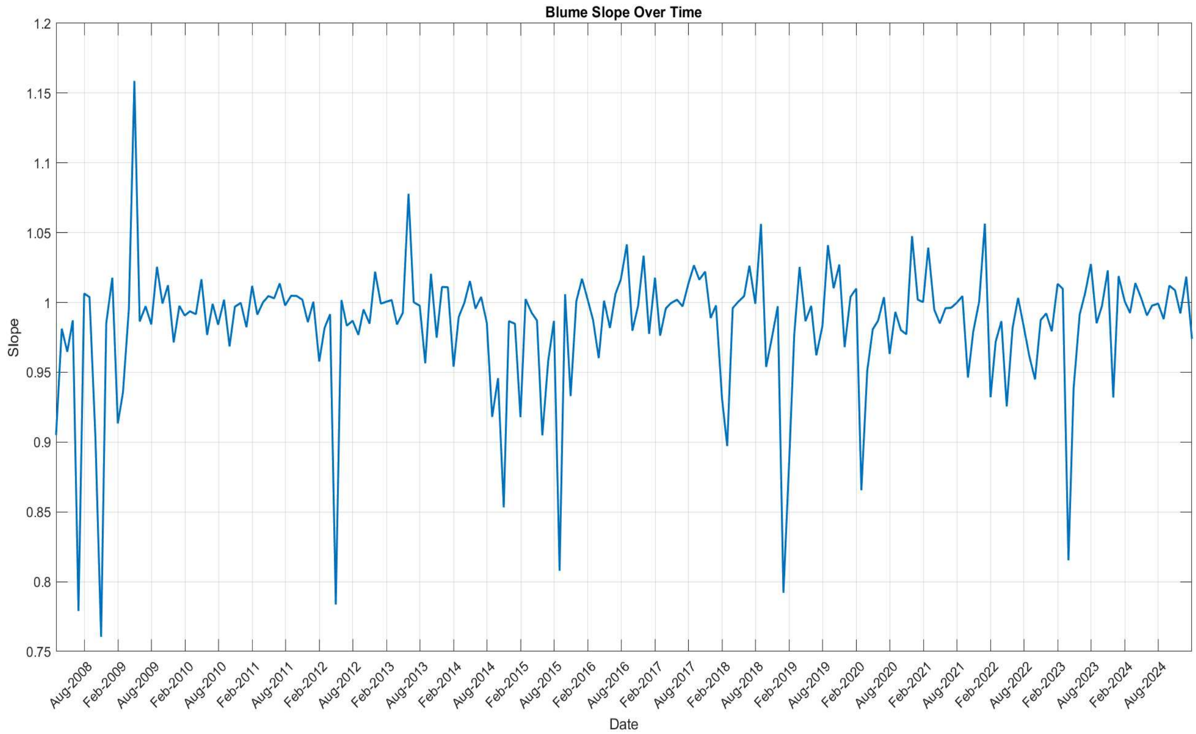

Figure 5 shows Blume slope for total period

The Blume coefficients are computed at a monthly frequency but for clarification, time axis in the graph is shown as a six-month interval. Time series of Blume slope coefficients shows that values generally are close to 1 and indicates that high degree of persistence in the beta estimates. Overall, the values suggest that betas evolve slowly over time causing minimal adjustment with Blume betas for total period.

The next discussion will be made on covid period from March. 28,2020 to March.30,2022 for analyzing how different beta choice impact on portfolio performance in terms of Sharpe Ratio and cumulative Returns of portfolio. Portfolio performance is evaluated over the Covid period only. However, portfolio optimization at each month is based on 36 rolling month which includes observations from pre-Covid period especially in the beginning of the subperiod.

Table 10: Out-of-Sample Sharpe Ratio, Portfolio Mean Return and Standard Deviation for Subperiod

Models	Out of Sample Sharpe Ratio	Portfolio Mean Return	Portfolio Standard Deviation
Raw Beta	0.3084	0.0361	0.1106
Vasicek Beta	0.2629	0.0235	0.0820
Blume Beta	0.3158	0.0366	0.1094

Table 10 evaluates the portfolio performance in terms of the Sharpe ratio for the pandemic period from March 2020 to March 2022 which includes recover period for pandemic. From table 10 it can be seen that Sharpe Ratio for all beta models have been increased.

Another important note is that during the covid period, portfolio standard deviation of raw beta and Blume beta is increased while portfolio standard deviation of Vasicek beta is decreased. However, proportion of portfolio mean return increased more than portfolio standard deviation for all models compared to total period. The increase in the Sharpe ratios during pandemic period mainly attributed to changes in optimal portfolio weights which is driven by changes in the beta estimates. With raw beta model, higher dispersion in the beta estimates leads to greater dispersion in the expected returns and optimization leads to weights to be more concentrated on high beta stocks. These stocks experienced higher realized returns during covid recovery period. Since, increase in portfolio return exceeds increase in the risk, Sharpe ratio value increased compared to full period. With Vasicek model, shrinkage of beta estimates leads to reduced dispersion in the betas and leads to more balanced weights portfolios.

Therefore, portfolio become less exposure to high beta stocks compared to raw beta which optimal portfolio weights align less with realized returns of stocks. With increase in the realized return at the stock level which is the same for all beta models with strong shrinkage of beta estimates toward the cross-sectional mean, portfolio risk decreased however, Vasicek captures smaller increase in portfolio return compared to raw beta.

Blume beta has slightly higher Sharpe ratio than raw beta in subperiod. Blume apply mean reversion from monthly linear cross-sectional regression; therefore, Blume model can stabilize beta estimates without distorting the cross -sectional differences systematic risk across stocks compared to Vasicek model so Blume provides better portfolio risk-return tradeoff than Vasicek but very close to raw beta.

Additionally, from full Covid period, to see more clearly how the Sharpe ratio affected during the initial phase of Covid period, table 11 illustrates the out of sample Sharpe ratios for only beginning of pandemic period from 28 December 2019 to 30 April 2020.

Table 11: Out-of-Sample Sharpe Ratio, Portfolio Mean Return and Standard Deviation for Initial Phase of Covid Period

Models	Out of Sample Sharpe Ratio	Mean Portfolio Return	Portfolio Standard Deviation
Raw Beta	-0.2876	-0.0456	0.1655
Vasicek Beta	-0.2415	-0.0363	0.1585
Blume Beta	-0.2859	-0.0440	0.1610

From table 11 it can be seen that, at the beginning of Covid period out of sample Sharpe ratio become negative value. This is mainly due to decline in portfolio mean return combined with an increase in portfolio standard deviation, showing higher market uncertainty during initial phase of covid period compared to recovery and full period.

Although all models exhibit negative Sharpe ratios during the initial Covid 19 shock, Vasicek adjusted beta performs relatively better than other models. As its shrinkage approach decreases estimation noise in OLS estimation beta and more balanced portfolio weights leads to portfolio become less exposed to estimation errors and large optimal weights misallocations.

Next discussion will correspond to analyzing cumulative portfolio returns under the raw, Blume and Vasicek beta. Figure 6 shows evaluation of cumulative portfolio returns from beginning to end of the covid period.

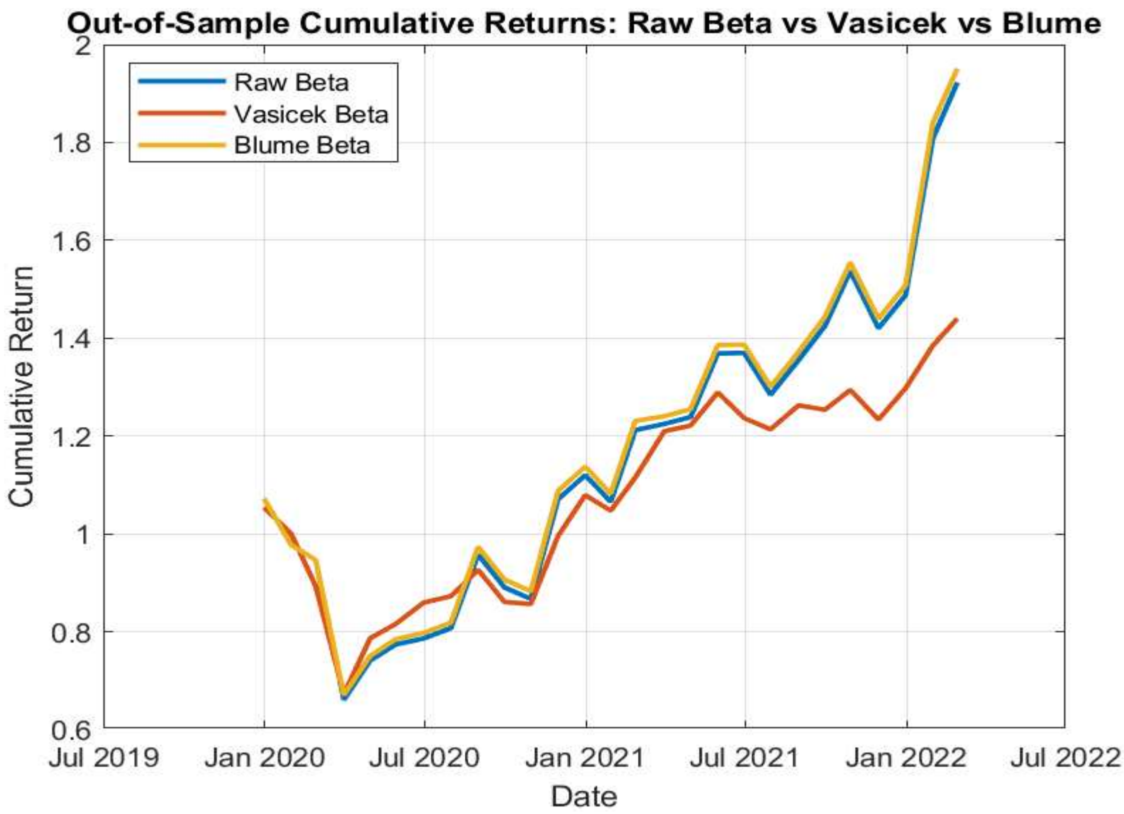

Figure 6 shows cumulative portfolio returns under the raw, Blume and Vasicek

Figure (6) displays evaluation of cumulative returns of portfolio for subperiod under the raw, Vasicek and Blume methods. Firstly, at the beginning of Covid period cumulative portfolio returns decreased but later in the Covid period, with recovery, cumulative portfolio returns increased.

Blume and raw beta follow very similar paths and Blume slightly outperformed the raw beta while Vasicek beta generally performs worse than raw and Blume beta in terms of cumulative returns except beginning of the covid period.

At the beginning of the covid period, estimation of beta with OLS became significantly noisy so historical estimation of beta becomes unreliable when structural changes occur in the market. Since Vasicek applies strong shrinkage when estimation error is high which pulls extreme beta estimates toward cross-sectional mean leading to more robust expected return estimates so optimal weight allocation for stocks in the sample become more robust which gives better realized portfolio cumulative returns. However, recovery period of pandemic Vasicek method underestimates the optimal weights of high beta stocks by shrinking their beta toward the mean leading to more biased expected returns and not robust optimal portfolio weights, so it dampens realized cumulative returns compared to raw and Blume method.

With Blume beta slope coefficient obtained from cross-sectional regression of stocks, still almost 1 as total period because forecast horizon is 1 month, the regression Blume lag is also 1 month prior so difference between raw and Blume beta remains very close to each other. As a result, there is no improvement obtained under both adjustment methods compared to raw beta for portfolio performance for subperiod. The slope of Blume coefficients evaluation over subperiod can be illustrated by figure (7)

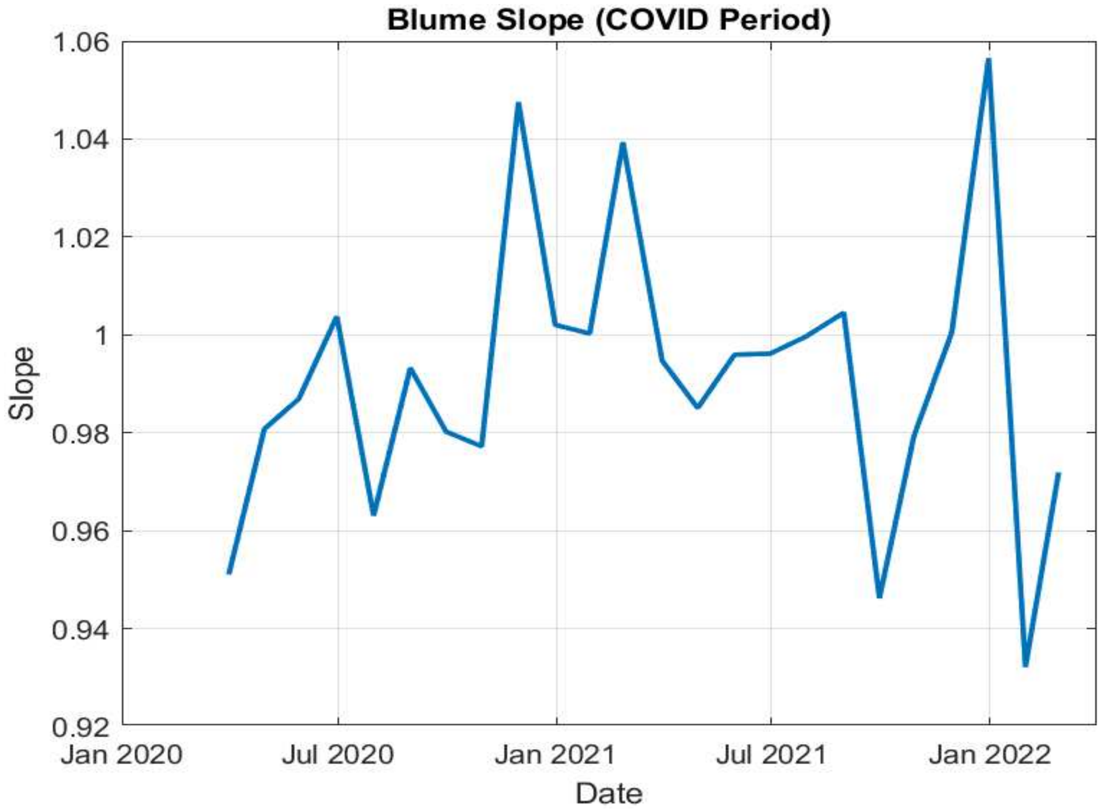

Figure 7 shows the slope of Blume for pandemic period

From figure (7) it can be clearly seen that Blume slope fluctuates around the value of 1. As a result, betas are persistent where future betas are closely dependent on the current beta; thus, Blume adjustment provides almost same values as raw beta in terms of Sharpe ratio and cumulative performance due to optimal portfolio weights for both betas are close to each other. Because of evaluating dynamic portfolio performance, forecast horizon is set to one month ahead therefore, Blume lag is also set to one month. With this framework, study was obtained that mean reversion of the Blume slope coefficient remains close to raw beta. Analyzing portfolio performance of raw and Blume beta yields similar optimal weights between models and this leads to same results for out of sample Sharpe ratio and cumulative portfolio returns. In addition to Blume beta, we can consider the fixed Blume coefficient which was found from the Marshall Blume where Slope is 0.67 and intercept is 0.33 to compare with Blume beta obtained from methodology part. This comparison allows us to assess whether the fixed coefficients provide similar or better portfolio performance than the data driven specific Blume coefficients.

Figure (8) now includes also the fixed Blume beta for analyzing out sample cumulative returns of portfolio for subperiod and later period. This enables us to examine how the fixed-coefficient Blume beta differs from raw beta, particularly when the data-driven Blume beta follows a similar pattern to raw beta due to monthly updates of the Blume coefficients used in dynamic optimal weight allocation for evaluating dynamic portfolio performance.

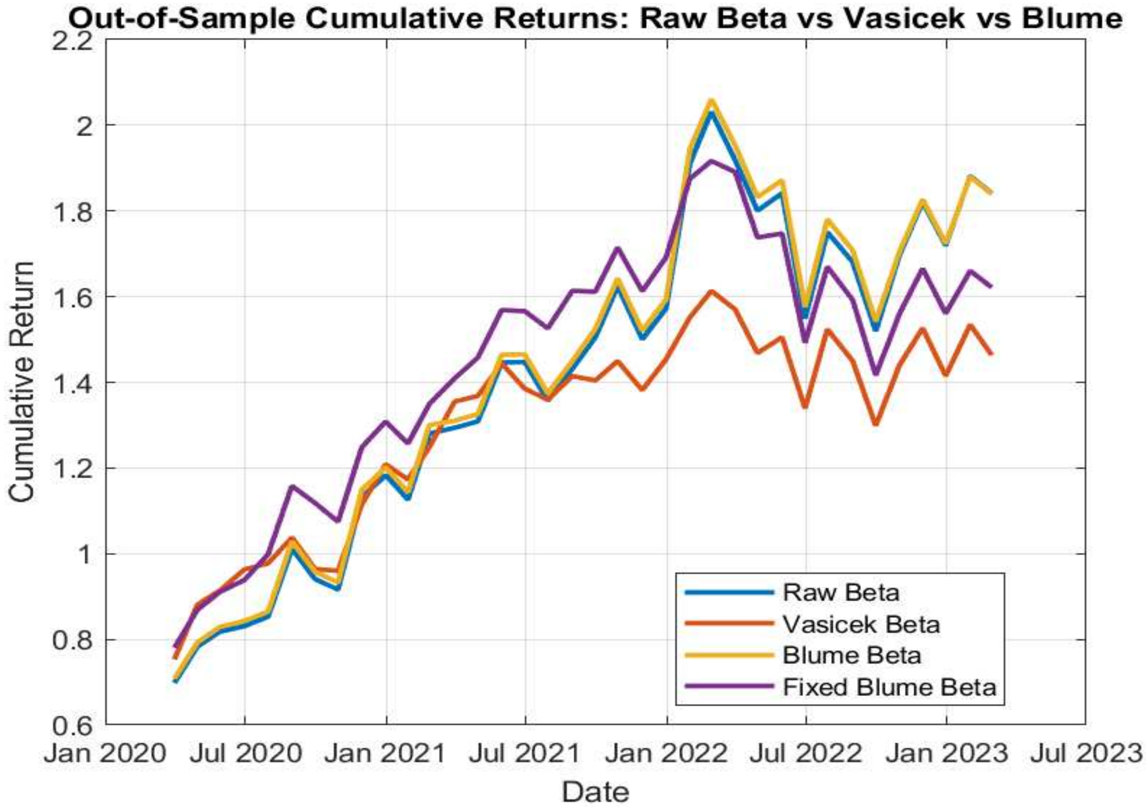

Figure 8 shows Cumulative portfolio returns under the raw, Blume, Vasicek and Blume beta with fixed coefficients.

From figure (8) Blume beta with fixed coefficients, stronger mean reversion leads to cumulative returns of portfolio deviates more than from those based on raw beta. In the beginning of period pandemic, both fixed Blume and Vasicek beta gives better cumulative portfolio returns than raw beta and Blume beta, as their stronger mean reversion improves beta estimates in terms of decreasing magnitude of high risk betas stocks and increasing magnitude of low risk beta stocks therefore it provides more accurate expected return and leads to portfolio weights that aligns with realized stock returns. However, in the later period fixed Blume beta and Vasicek beta underperforms relative raw beta and Blume beta as both impose stronger mean reversion, with Vasicek applying more shrinkage leading to excessive shrinkage of high betas toward the mean and upward adjustment of low betas. This causes bias in expected return estimates and leads to optimal portfolio weights do not align with realized stocks returns so low values for out of sample cumulative portfolio returns.

Overall, it can be stated that adjustment models have the better forecasting quality of the beta estimators compared to raw beta and Blume gives the highest forecasting quality of the beta estimators in the study. However, for portfolio performance evaluation based on the dynamic portfolio allocation, Vasicek and Blume fail to perform better than raw beta. With Vasicek correct sampling error in OLS whereas Blume method applies mean reversion to correct over and underestimation of beta in OLS. Since both adjustment methods reduce the dispersion of beta estimates, they improve statistical forecast accuracy. However, this improvement does not translate into higher portfolio performance, except during the initial phase of the COVID-19 period.

The Vasicek method applies a shrinkage approach, pulling high-beta stocks downward and low-beta stocks upward toward the cross-sectional mean. Also, the Blume method adjusts high and low betas of stocks toward one through cross-sectional regression between past and current beta estimates of stocks. Therefore, high-beta stocks, which are associated with higher expected returns under the CAPM framework, receive lower portfolio weights, while low-beta stocks receive relatively higher weights. These optimal weights allocation reduces the portfolio’s exposure to high-return opportunities and limits its ability to achieve higher realized portfolio returns compared to raw beta. Furthermore, when Blume coefficients are updated on a one-month-ahead for dynamic portfolio performance, it does not deviate from raw beta in portfolio allocation. This is because the estimated coefficients are typically close to one, resulting in only minor adjustments to raw beta. In contrast, when fixed Blume coefficients are applied, the adjustment behaves more similarly to the Vasicek method. As a result, better statistical forecast accuracy does not yield better portfolio performance, and this finding is consistent with the recent papers.

Both papers suggested that low statistical forecast accuracy (RMSE and MSE) does yield high portfolio performance mainly due to portfolio decisions depend primarily on the ranking and timing of expected returns rather than their exact magnitude. Ranking refers to the ability of a model to correctly order assets according to their expected returns, while timing refers to identifying periods in which returns are relatively high or low. Therefore, even if a model exhibits large forecast errors in terms of RMSE or MSE, it may still achieve strong portfolio performance if it correctly captures the relative ordering and directional signals of returns. (Lee et al., 2024; Cenesizoglu & Timmermann, 2012).

Chapter 6: Further Research

In this research, only 36-month rolling window is used over the entire period. Future research could consider alternative rolling window lengths, such as 12-month or 24-month periods to examine how shorter estimation horizons influence forecast accuracy of betas and portfolio performance. Shorter windows may better capture recent market dynamics but could also introduce greater estimation noise.

In addition, future research could analyze fundamental beta models, which estimate the risk of firms based on a combination of firm-specific fundamentals and market-based characteristics of their stocks, such as dividend payout ratios (dividend divided by earnings), asset growth (annual change in total assets), leverage (total debt divided by total assets) and stock volatility. This approach may improve beta forecast accuracy and portfolio performance, as it incorporates firm-specific fundamental information's that is not captured directly by adjusted beta models.

Further research could also incorporate multifactor asset pricing models, such as the Fama-French three factor model, which extends the CAPM by including additional risk factors such as, Size factor (small-cap stocks vs large-cap stocks), Value factor (High book-to-market vs low book-to-market) and market factor to evaluate their impact on beta forecast accuracy and portfolio performance.

Chapter 7: Conclusion

This study analyzed the performance of raw, Blume and Vasicek beta estimators in terms of both forecast accuracy and portfolio performance. Using a rolling 36-month estimation window, the analysis compared the effectiveness of these estimation beta methods for S&P 500 stocks in predicting future betas and their implications for the portfolio construction. The aim of this study is to find out to what extent the adjustment models can improve the forecasting quality of stock beta (raw beta) and mean-variance portfolio optimization.

For the total period and pandemic period, the empirical analysis shows that in terms of the forecast accuracy, the Mean Squared Error (MSE), the Root MSE (RMSE), the Mean Absolute Error (MAE) and Theil's uncertainty coefficients indicate that the adjusted beta estimators outperform the raw beta, providing more accurate beta forecasts. In addition, the decomposition of MSE into bias, inefficiency and random noise components further supports the superiority of adjusted beta estimators.

For the total period and pandemic period, in terms of portfolio performance, the results based on the Sharpe ratio and cumulative returns suggest that adjusted beta estimators could not outperform the raw beta. However, during the early stage of the pandemic period Vasicek and Blume with fixed coefficients yield higher cumulative portfolio returns compared to raw beta.

In summary, the present analysis shows that adjusted beta estimators improve the forecast accuracy of beta, with the Blume method providing the best performance. However, this improvement does not lead to better dynamic portfolio performance, as dynamic portfolios constructed using optimal portfolio weights from raw and Blume betas produce equivalent results, while Vasicek method performs worse. As a result, achieving the lowest forecast error does not lead to higher economic performance in terms of portfolio returns.

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